

# Package ‘admix’

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**Title** Package Admix for Admixture (aka Contamination) Models

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**Description** Implements techniques to estimate the unknown quantities related to two-component admixture models, where the two components can belong to any distribution (note that in the case of multinomial mixtures, the two components must belong to the same family). Estimation methods depend on the assumptions made on the unknown component density (see Bordes and Vandekerkhove (2010) <[doi:10.3103/S1066530710010023](https://doi.org/10.3103/S1066530710010023)>; Patra and Sen (2016) <[doi:10.1111/rssb.12148](https://doi.org/10.1111/rssb.12148)>; Milhaud, Pommeret, Salhi and Vandekerkhove (2022) <[doi:10.1016/j.jspi.2021.05.010](https://doi.org/10.1016/j.jspi.2021.05.010)>). In practice, one can estimate both the mixture weight and the unknown component density in a wide variety of frameworks. On top of that, hypothesis tests can be performed in one and two-sample contexts to test the unknown component density (see Milhaud, Pommeret, Salhi, Vandekerkhove (2023)). Finally, clustering of unknown mixture components is also feasible in a K-samples setting.

**License** GPL (>= 3)

**URL** <https://github.com/XavierMilhaud/admix>

**BugReports** <https://github.com/XavierMilhaud/admix/issues>

**Encoding** UTF-8

**LazyData** true

**RoxygenNote** 7.2.3

**Imports** base, cubature, fdrtool, graphics, Iso, MASS, methods, orthopolynom, pracma, Rcpp, stats, utils

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**Depends** R (>= 2.10)

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admix_clustering	<i>Clustering of K populations following admixture models</i>
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**Description**

Create clusters on the unknown components related to the K populations following admixture models. Based on the K-sample test using Inversion - Best Matching (IBM) approach, see 'Details' below for further information.

**Usage**

```
admix_clustering(
  samples = NULL,
  n_sim_tab = 100,
  comp.dist = NULL,
  comp.param = NULL,
  tabul.dist = NULL,
  tune.penalty = FALSE,
  conf.level = 0.95,
  parallel = FALSE,
  n_cpu = 2,
  echo = TRUE
)
```

**Arguments**

- |           |  |
|-----------|--|
| samples   | A list of the K observed samples to be clustered, all following admixture distributions.                                 |
| n_sim_tab | Number of simulated gaussian processes used in the tabulation of the inner convergence distribution in the IBM approach. |

comp.dist	A list with 2*K elements corresponding to the component distributions (specified with R native names for these distributions) involved in the K admixture models. Elements, grouped by 2, refer to the unknown and known components of each admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows with K = 3: list(f1 = NULL, g1 = 'norm', f2 = NULL, g2 = 'norm', f3 = NULL, g3 = 'rnorm').
comp.param	A list with 2*K elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Elements, grouped by 2, refer to the parameters of unknown and known components of each admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows (with K = 3): list(f1 = NULL, g1 = list(mean=0,sd=1), f2 = NULL, g2 = list(mean=3,sd=1.1), f3 = NULL, g3 = list(mean=-2,sd=0.6)).
tabul.dist	Only useful for comparisons of detected clusters at different confidence levels. Is a list of the tabulated distributions of the stochastic integral for each cluster previously detected.
tune.penalty	A boolean that allows to choose between a classical penalty term or an optimized penalty embedding some tuning parameters (automatically optimized) for k-sample tests used within the clustering procedure. Optimized penalty is particularly useful for low sample size.
conf.level	The confidence level of the K-sample test used in the clustering procedure.
parallel	(default to FALSE) Boolean to indicate whether parallel computations are performed (speed-up the tabulation).
n_cpu	(default to 2) Number of cores used when parallelizing.
echo	(default to TRUE) Display the remaining computation time.

### Details

See the paper at the following HAL weblink: <https://hal.science/hal-04129130>

### Value

A list with eleven elements: 1) the number of populations studied; 2) the number of detected clusters; 3) the list of p-values for each test performed; 4) the cluster affiliation for each population; 5) the chosen confidence level of statistical tests; 6) the cluster components; 7) the size of clusters; 8) the estimated weights of the unknown component distributions inside each cluster (remind that estimated weights are consistent only if unknown components are tested to be identical); 9) the matrix of pairwise discrepancies across populations; 10) the tabulated distributions used for statistical tests; 11) the initial call.

### Author(s)

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## Examples

```
## Simulate data (chosen parameters indicate 2 clusters (populations (1,3), and (2,4)):
list.comp <- list(f1 = "gamma", g1 = "exp",
                 f2 = "gamma", g2 = "exp",
                 f3 = "gamma", g3 = "gamma",
                 f4 = "gamma", g4 = "exp")
list.param <- list(f1 = list(shape = 16, rate = 4), g1 = list(rate = 1/3.5),
                  f2 = list(shape = 14, rate = 2), g2 = list(rate = 1/5),
                  f3 = list(shape = 16, rate = 4), g3 = list(shape = 12, rate = 2),
                  f4 = list(shape = 14, rate = 2), g4 = list(rate = 1/7))
A.sim <- rsimmix(n=2600, unknownComp_weight=0.8, comp.dist = list(list.comp$f1,list.comp$g1),
                comp.param = list(list.param$f1, list.param$g1))$mixt.data
B.sim <- rsimmix(n=3000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
                comp.param = list(list.param$f2, list.param$g2))$mixt.data
C.sim <- rsimmix(n=3500, unknownComp_weight=0.6, comp.dist = list(list.comp$f3,list.comp$g3),
                comp.param = list(list.param$f3, list.param$g3))$mixt.data
D.sim <- rsimmix(n=4800, unknownComp_weight=0.5, comp.dist = list(list.comp$f4,list.comp$g4),
                comp.param = list(list.param$f4, list.param$g4))$mixt.data
## Look for the clusters:
list.comp <- list(f1 = NULL, g1 = "exp",
                 f2 = NULL, g2 = "exp",
                 f3 = NULL, g3 = "gamma",
                 f4 = NULL, g4 = "exp")
list.param <- list(f1 = NULL, g1 = list(rate = 1/3.5),
                  f2 = NULL, g2 = list(rate = 1/5),
                  f3 = NULL, g3 = list(shape = 12, rate = 2),
                  f4 = NULL, g4 = list(rate = 1/7))
clusters <- admix_clustering(samples = list(A.sim,B.sim,C.sim,D.sim), n_sim_tab = 30,
                             comp.dist = list.comp, comp.param = list.param,
                             tabul.dist = NULL, tune.penalty = TRUE, conf.level = 0.95,
                             parallel = TRUE, n_cpu = 2, echo = FALSE)

clusters
```

---

 admix\_estim

*Estimate the unknown parameters of the admixture model(s) under study*

---

## Description

Estimate the component weights, the location shift parameter (in case of a symmetric unknown component density), and the unknown component distribution using different estimation techniques. We remind that the  $i$ -th admixture model has probability density function (pdf)  $l_i$  such that:  $l_i = p_i * f_i + (1-p_i) * g_i$ , where  $g_i$  is the known component density. The unknown quantities  $p_i$  and  $f_i$  then have to be estimated.

**Usage**

```
admix_estim(
  samples = NULL,
  sym.f = FALSE,
  est.method = c("PS", "BVdk", "IBM"),
  comp.dist = NULL,
  comp.param = NULL
)
```

**Arguments**

<code>samples</code>	A list of the $K$ samples to be studied, all following admixture distributions.
<code>sym.f</code>	A boolean indicating whether the unknown component densities are assumed to be symmetric or not.
<code>est.method</code>	The estimation method to be applied. Can be one of 'BVdk' (Bordes and Vandekerkhove estimator), 'PS' (Patra and Sen estimator), or 'IBM' (Inversion Best-Matching approach). The same estimation method is performed on each sample. Important note: estimation by 'IBM' is unbiased only under $H_0$ , meaning that choosing this method requires to perform previously the test hypothesis between the pairs of samples. For further details, see section 'Details' below.
<code>comp.dist</code>	A list with $2*K$ elements corresponding to the component distributions (specified with R native names for these distributions) involved in the $K$ admixture models. Elements, grouped by 2, refer to the unknown and known components of each admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows with $K = 3$ : <code>list(f1 = NULL, g1 = 'norm', f2 = NULL, g2 = 'norm', f3 = NULL, g3 = 'norm')</code> .
<code>comp.param</code>	A list with $2*K$ elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Elements, grouped by 2, refer to the parameters of unknown and known components of each admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows (with $K = 3$ ): <code>list(f1 = NULL, g1 = list(mean=0,sd=1), f2 = NULL, g2 = list(mean=3,sd=1.1), f3 = NULL, g3 = list(mean=-2,sd=0.6))</code> .

**Details**

For further details on the different estimation techniques, see i) IBM approach at <https://hal.science/hal-03201760> ; ii) Patra and Sen estimator: Patra, R.K. and Sen, B. (2016); Estimation of a Two-component Mixture Model with Applications to Multiple Testing; JRSS Series B, 78, pp. 869–893. ; iii) BVdk estimator: Bordes, L. and Vandekerkhove, P. (2010); Semiparametric two-component mixture model when a component is known: an asymptotically normal estimator; Math. Meth. Stat.; 19, pp. 22–41.

**Value**

A list containing the estimated weight of every unknown component distribution among admixture samples.

**Author(s)**

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**Examples**

```
##### On a simulated example to see whether the true parameters are well estimated.
list.comp <- list(f1 = "norm", g1 = "norm",
                f2 = "norm", g2 = "norm")
list.param <- list(f1 = list(mean = 0, sd = 1), g1 = list(mean = 2, sd = 0.7),
                  f2 = list(mean = 0, sd = 1), g2 = list(mean = -3, sd = 1.1))
## Simulate data:
sim1 <- rsimmix(n = 2100, unknownComp_weight = 0.8, comp.dist = list(list.comp$f1, list.comp$g1),
               comp.param = list(list.param$f1, list.param$g1))$mixt.data
sim2 <- rsimmix(n = 2000, unknownComp_weight = 0.85, comp.dist = list(list.comp$f2, list.comp$g2),
               comp.param = list(list.param$f2, list.param$g2))$mixt.data
## Estimate the mixture weights of the admixture models:
list.comp <- list(f1 = NULL, g1 = "norm",
                 f2 = NULL, g2 = "norm")
list.param <- list(f1 = NULL, g1 = list(mean = 2, sd = 0.7),
                  f2 = NULL, g2 = list(mean = -3, sd = 1.1))
estim <- admix_estim(samples = list(sim1, sim2), sym.f = TRUE, est.method = 'IBM',
                    comp.dist = list.comp, comp.param = list.param)
```

---

admix\_test

*Hypothesis test between unknown components of the admixture models under study*

---

**Description**

Perform hypothesis test between unknown components of a list of admixture models, where we remind that the  $i$ -th admixture model has probability density function (pdf)  $l_i$  such that:  $l_i = p_i * f_i + (1-p_i) * g_i$ , with  $g_i$  the known component density. The unknown quantities  $p_i$  and  $f_i$  are thus estimated, leading to the test given by the following null and alternative hypothesis:  $H_0: f_i = f_j$  for all  $i \neq j$  against  $H_1$ : there exists at least  $i \neq j$  such that  $f_i$  differs from  $f_j$ . The test can be performed using two methods, either the comparison of coefficients obtained through polynomial basis expansions of the component densities, or by the inner-convergence property obtained using the IBM approach. See 'Details' below for further information.

**Usage**

```

admix_test(
  samples = NULL,
  sym.f = FALSE,
  test.method = c("Poly", "ICV"),
  sim_U = NULL,
  n_sim_tab = 50,
  comp.dist = NULL,
  comp.param = NULL,
  support = c("Real", "Integer", "Positive", "Bounded.continuous"),
  ICV_tunePenalty = TRUE,
  conf.level = 0.95,
  parallel = FALSE,
  n_cpu = 2
)

```

**Arguments**

<code>samples</code>	A list of the K samples to be studied, all following admixture distributions.
<code>sym.f</code>	A boolean indicating whether the unknown component densities are assumed to be symmetric or not.
<code>test.method</code>	The testing method to be applied. Can be either 'Poly' (polynomial basis expansion) or 'ICV' (inner convergence from IBM). The same testing method is performed between all samples. In the one-sample case, only 'Poly' is available and the test is a gaussianity test. For further details, see section 'Details' below.
<code>sim_U</code>	(Used only with 'ICV' testing method, otherwise useless) Random draws of the inner convergence part of the contrast as defined in the IBM approach (see 'Details' below).
<code>n_sim_tab</code>	(Used only with 'ICV' testing method, otherwise useless) Number of simulated gaussian processes used in the tabulation of the inner convergence distribution in the IBM approach.
<code>comp.dist</code>	A list with 2*K elements corresponding to the component distributions (specified with R native names for these distributions) involved in the K admixture models. Elements, grouped by 2, refer to the unknown and known components of each admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows with K = 3: <code>list(f1 = NULL, g1 = 'norm', f2 = NULL, g2 = 'norm', f3 = NULL, g3 = 'norm')</code> .
<code>comp.param</code>	A list with 2*K elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Elements, grouped by 2, refer to the parameters of unknown and known components of each admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows (with K = 3): <code>list(f1 = NULL, g1 = list(mean=0,sd=1), f2 = NULL, g2 = list(mean=3,sd=1.1), f3 = NULL, g3 = list(mean=-2,sd=0.6))</code> .



support	(Potentially used with 'Poly' testing method, otherwise useless) The support of the observations; one of "Real", "Integer", "Positive", or "Bounded.continuous".
ICV_tunePenalty	(default to TRUE) Boolean used to tune the penalty term in the k-sample test (k=2,3,...,K) when using Inversion Best Matching (IBM) approach coupled to Inner ConVergence (ICV) property. Particularly useful when studying unbalanced samples (in terms of sample size) or small-sized samples.
conf.level	The confidence level of the K-sample test.
parallel	(default to FALSE) Boolean indicating whether parallel computations are performed (speed-up the tabulation).
n_cpu	(default to 2) Number of cores used when parallelizing.

### Details

For further details on hypothesis techniques, see i) Inner convergence through IBM approach at <https://hal.science/hal-03201760> ; ii) Polynomial expansions at 'False Discovery Rate model Gaussianity test' (EJS, Pommeret & Vanderkerkhove, 2017), or 'Semiparametric two-sample admixture components comparison test: the symmetric case' (JSPI, Milhaud & al., 2021).

### Value

A list containing the decision of the test (reject or not), the confidence level at which the test is performed, the p-value of the test, and the value of the test statistic (following a chi2 distribution with one degree of freedom under the null).

### Author(s)

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### Examples

```
##### On a simulated example, with 1 sample (gaussianity test):
list.comp <- list(f1 = "norm", g1 = "norm")
list.param <- list(f1 = list(mean = 0, sd = 1), g1 = list(mean = 2, sd = 0.7))
## Simulate data:
sim1 <- rsimmix(n = 300, unknownComp_weight = 0.85, comp.dist = list(list.comp$f1, list.comp$g1),
               comp.param = list(list.param$f1, list.param$g1))$mixt.data
## Perform the test hypothesis:
list.comp <- list(f1 = NULL, g1 = "norm")
list.param <- list(f1 = NULL, g1 = list(mean = 2, sd = 0.7))
gaussTest <- admix_test(samples = list(sim1), sym.f = TRUE, test.method = 'Poly', sim_U = NULL,
                       n_sim_tab = 50, comp.dist = list.comp, comp.param = list.param,
                       support = "Real", conf.level = 0.95, parallel = FALSE, n_cpu = 2)
```

---

allGalaxies	<i>Four galaxies (Carina, Sextans, Sculptor, Fornax) measurements of heliocentric velocities from SIMBAD astronomical database</i>
-------------	--

---

**Description**

Four galaxies (Carina, Sextans, Sculptor, Fornax) measurements of heliocentric velocities from SIMBAD astronomical database

**Usage**

allGalaxies

**Format**

An evolving data frame of velocities for 4 dSph galaxies; namely Carina, Sextans, Sculptor and Fornax. Currently contains 8,862 rows and 3 columns, with information on:

**Target** Target identification; Galaxy-ID number

**HV** Weighted mean Heliocentric rest frame velocity

**Name** The name of the galaxy

**Source**

[https://vizier.u-strasbg.fr/viz-bin/VizieR-3?-source=J/AJ/137/3100/stars&-out.max=50&-out.form=HTML%20Table&-out.add=\\_r&-out.add=\\_RAJ,\\_DEJ&-out.add=\\_RA%2a-c.eq,\\_DE%2a-c.eq&-sort=\\_r&-oc.form=sexa](https://vizier.u-strasbg.fr/viz-bin/VizieR-3?-source=J/AJ/137/3100/stars&-out.max=50&-out.form=HTML%20Table&-out.add=_r&-out.add=_RAJ,_DEJ&-out.add=_RA%2a-c.eq,_DE%2a-c.eq&-sort=_r&-oc.form=sexa)

---

BVdk_contrast	<i>Contrast as defined in Bordes &amp; Vandekerkhove (2010)</i>
---------------	---

---

**Description**

Compute the contrast as defined in Bordes & Vandekerkhove (2010) (see below in section 'Details'), needed for optimization purpose. Remind that one considers an admixture model with symmetric unknown density, i.e.  $l(x) = p*f(x-\mu) + (1-p)*g(x)$ , where  $l$  denotes the probability density function (pdf) of the mixture with known component pdf  $g$ ,  $p$  is the unknown mixture weight,  $f$  relates to the unknown symmetric component pdf  $f$ , and  $\mu$  is the location shift parameter.

**Usage**

BVdk\_contrast(param, data, h, comp.dist, comp.param)

**Arguments**

param	Numeric vector of two elements, corresponding to the two parameters (first the unknown component weight, and then the location shift parameter of the symmetric unknown component distribution).
data	Numeric vector of observations following the admixture model given by the pdf $l$ .
h	Width of the window used in the kernel estimations.
comp.dist	A list with two elements corresponding to component distributions (specified with R native names for these distributions) involved in the admixture model. Unknown elements must be specified as 'NULL' objects, e.g. when 'f' is unknown: <code>list(f=NULL, g='norm')</code> .
comp.param	A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Unknown elements must be specified as 'NULL' objects, e.g. if 'f' is unknown: <code>list(f=NULL, g=list(mean=0,sd=1))</code> .

**Details**

The contrast is defined in Bordes, L. and Vandekerkhove, P. (2010); Semiparametric two-component mixture model when a component is known: an asymptotically normal estimator; *Math. Meth. Stat.*; 19, pp. 22–41.

**Value**

The value of the contrast.

**Author(s)**

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**Examples**

```
## Simulate data:
comp.dist <- list(f = 'norm', g = 'norm')
comp.param <- list(f = list(mean = 3, sd = 0.5), g = list(mean = 0, sd = 1))
data1 <- rsimmix(n = 1000, unknownComp_weight = 0.6, comp.dist, comp.param)[['mixt.data']]
## Compute the contrast value for some given parameter vector in real-life framework:
comp.dist <- list(f = NULL, g = 'norm')
comp.param <- list(f = NULL, g = list(mean = 0, sd = 1))
BVdk_contrast(c(0.3,2), data1, density(data1)$bw, comp.dist, comp.param)
```

---

 BVdk\_contrast\_gradient

*Gradient of the contrast as defined in Bordes & Vandekerkhove (2010)*


---

### Description

Compute the gradient of the contrast as defined in Bordes & Vandekerkhove (2010) (see below in section 'Details'), needed for optimization purpose. Remind that one considers an admixture model, i.e.  $l = p*f + (1-p)*g$  ; where  $l$  denotes the probability density function (pdf) of the mixture with known component pdf  $g$ ,  $p$  is the unknown mixture weight, and  $f$  relates to the unknown symmetric component pdf  $f$ .

### Usage

```
BVdk_contrast_gradient(param, data, h, comp.dist, comp.param)
```

### Arguments

param	A numeric vector with two elements corresponding to the parameters to be estimated. First the unknown component weight, and second the location shift parameter of the symmetric unknown component distribution.
data	A vector of observations following the admixture model given by the pdf $l$ .
h	The window width used in the kernel estimations.
comp.dist	A list with two elements corresponding to component distributions (specified with R native names for these distributions) involved in the admixture model. Unknown elements must be specified as 'NULL' objects, e.g. when 'f' is unknown: <code>list(f=NULL, g='norm')</code> .
comp.param	A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Unknown elements must be specified as 'NULL' objects, e.g. if 'f' is unknown: <code>list(f=NULL, g=list(mean=0,sd=1))</code> .

### Details

The contrast is defined in Bordes, L. and Vandekerkhove, P. (2010); Semiparametric two-component mixture model when a component is known: an asymptotically normal estimator; *Math. Meth. Stat.*; 19, pp. 22–41.

### Value

A numeric vector composed of the two partial derivatives w.r.t. the two parameters on which to optimize the contrast.

### Author(s)

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## Examples

```
## Simulate data:
comp.dist <- list(f = 'norm', g = 'norm')
comp.param <- list(f = list(mean = 3, sd = 0.5), g = list(mean = 0, sd = 1))
data1 <- rsimmix(n = 1000, unknownComp_weight = 0.6, comp.dist, comp.param)[['mixt.data']]
## Compute the contrast gradient for some given parameter vector in real-life framework:
comp.dist <- list(f = NULL, g = 'norm')
comp.param <- list(f = NULL, g = list(mean = 0, sd = 1))
BVdk_contrast_gradient(c(0.3,2), data1, density(data1)$bw, comp.dist, comp.param)
```

---

BVdk_estimParam	<i>Estimation of the parameters in a two-component admixture model with symmetric unknown density</i>
-----------------	---

---

## Description

Estimation of the two parameters (mixture weight as well as location shift) in the admixture model with pdf:  $l(x) = p \cdot f(x - \mu) + (1 - p) \cdot g(x)$ ,  $x$  in  $\mathbb{R}$ , where  $g$  is the known component,  $p$  is the proportion and  $f$  is the unknown component with symmetric density. The localization shift parameter is thus denoted  $\mu$ , and the component weight  $p$ . See 'Details' below for further information.

## Usage

```
BVdk_estimParam(
  data,
  method = c("L-BFGS-B", "Nelder-Mead"),
  comp.dist,
  comp.param
)
```

## Arguments

data	The observed sample under study.
method	The method used throughout the optimization process, either 'L-BFGS-B' or 'Nelder-Mead' (see ?optim).
comp.dist	A list with two elements corresponding to component distributions (specified with R native names for these distributions) involved in the admixture model. Unknown elements must be specified as 'NULL' objects, e.g. when 'f' is unknown: list(f=NULL, g='norm').
comp.param	A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Unknown elements must be specified as 'NULL' objects, e.g. if 'f' is unknown: list(f=NULL, g=list(mean=0,sd=1)).

**Details**

Parameters are estimated by minimization of the contrast function, where the contrast is defined in Bordes, L. and Vandekerkhove, P. (2010); Semiparametric two-component mixture model when a component is known: an asymptotically normal estimator; Math. Meth. Stat.; 19, pp. 22–41.

**Value**

A numeric vector with the two estimated parameters (proportion first, and then location shift).

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
## Simulate data:
list.comp <- list(f = 'norm', g = 'norm')
list.param <- list(f = list(mean = -2, sd = 0.5),
                  g = list(mean = 0, sd = 1))
data1 <- rsimmix(n = 200, unknownComp_weight = 0.4, list.comp, list.param)[['mixt.data']]
## Perform the estimation of parameters in real-life:
list.comp <- list(f = NULL, g = 'norm')
list.param <- list(f = NULL, g = list(mean = 0, sd = 1))
BVdk_estimParam(data1, method = 'L-BFGS-B', list.comp, list.param)
```

---

BVdk\_ML\_varCov\_estimators

*Maximum Likelihood estimation of the variance of the unknown density variance estimator in an admixture model*

---

**Description**

Parametric estimation of the variance of the variance parameter in Bordes & Vandekerkhove (2010) setting, i.e. considering the admixture model with probability density function (pdf)  $l(x) = p \cdot f(x - \mu) + (1 - p) \cdot g$ , where  $g$  is the known component of the two-component mixture,  $p$  is the mixture proportion,  $f$  is the unknown component with symmetric density, and  $\mu$  is the location shift parameter. The estimation of the variance of the variance related to the density  $f$  is made by maximum likelihood optimization through the information matrix, with the assumption that the unknown  $f$  is gaussian.

**Usage**

```
BVdk_ML_varCov_estimators(data, hat_w, hat_loc, hat_var, comp.dist, comp.param)
```

**Arguments**

<code>data</code>	The observed sample under study.
<code>hat_w</code>	Estimate of the unknown component weight.
<code>hat_loc</code>	Estimate of the location shift parameter.
<code>hat_var</code>	Estimate of the variance of the symmetric density $f$ , obtained by plugging-in the previous estimates. See 'Details' below for further information.
<code>comp.dist</code>	A list with two elements corresponding to component distributions (specified with R native names for these distributions) involved in the admixture model. Unknown elements must be specified as 'NULL' objects, e.g. when 'f' is unknown: <code>list(f=NULL, g='norm')</code> .
<code>comp.param</code>	A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Unknown elements must be specified as 'NULL' objects, e.g. if 'f' is unknown: <code>list(f=NULL, g=list(mean=0,sd=1))</code> .

**Details**

Plug-in strategy is defined in Pommeret, D. and Vandekerkhove, P. (2019); Semiparametric density testing in the contamination model; *Electronic Journal of Statistics*, 13, pp. 4743–4793. The variance of the estimator variance of the unknown density  $f$  is needed in a testing perspective, since included in the variance of the test statistic. Other details about the information matrix can be found in Bordes, L. and Vandekerkhove, P. (2010); Semiparametric two-component mixture model when a component is known: an asymptotically normal estimator; *Math. Meth. Stat.*; 19, pp. 22–41.

**Value**

The variance of the estimator of the variance of the unknown component density  $f$ .

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
## Simulate data:
list.comp <- list(f = "norm", g = "norm")
list.param <- list(f = c(mean = 4, sd = 1), g = c(mean = 7, sd = 0.5))
sim.data <- rsimmix(n = 400, unknownComp_weight = 0.9, list.comp, list.param)$mixt.data
## Estimate mixture weight and location shift parameters in real-life:
list.comp <- list(f = NULL, g = "norm")
list.param <- list(f = NULL, g = c(mean = 7, sd = 0.5))
estim <- BVdk_estimParam(data = sim.data, method = "L-BFGS-B",
                        comp.dist = list.comp, comp.param = list.param)
## Estimation of the second-order moment of the known component distribution:
m2_knownComp <- mean(rnorm(n = 1000000, mean = 7, sd = 0.5)^2)
hat_s2 <- (1/estim[1]) * (mean(sim.data^2) - ((1-estim[1])*m2_knownComp)) - estim[2]^2
```

```
## Estimated variance of variance estimator related to the unknown symmetric component density:
BVdk_ML_varCov_estimators(data = sim.data, hat_w = estim[1], hat_loc = estim[2],
                           hat_var = hat_s2, comp.dist = list.comp, comp.param = list.param)
```

---

BVdk\_varCov\_estimators

*Estimation of the variance of the estimators in admixture models with symmetric unknown density.*

---

### Description

Semiparametric estimation of the variance of the estimators, i.e. the mixture weight  $p$  and the location shift parameter  $\mu$  considering the admixture model with probability density function  $l$ :  $l(x) = p \cdot f(x - \mu) + (1 - p) \cdot g(x)$ ,  $x$  in  $\mathbb{R}$ , where  $g$  is the known component of the two-component mixture,  $p$  is the unknown proportion,  $f$  is the unknown component density and  $\mu$  is the location shift. See 'Details' below for more information.

### Usage

```
BVdk_varCov_estimators(data, loc, p, comp.dist, comp.param)
```

### Arguments

<code>data</code>	The observed sample under study.
<code>loc</code>	The estimated location shift parameter, related to the unknown symmetric density.
<code>p</code>	The estimated unknown component weight.
<code>comp.dist</code>	A list with two elements corresponding to component distributions (specified with R native names for these distributions) involved in the admixture model. Unknown elements must be specified as 'NULL' objects, e.g. when 'f' is unknown: <code>list(f=NULL, g='norm')</code> .
<code>comp.param</code>	A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Unknown elements must be specified as 'NULL' objects, e.g. if 'f' is unknown: <code>list(f=NULL, g=list(mean=0,sd=1))</code> .

### Details

See formulas pp.28–30 in Appendix of Bordes, L. and Vandekerkhove, P. (2010); Semiparametric two-component mixture model when a component is known: an asymptotically normal estimator; *Math. Meth. Stat.*; 19, pp. 22–41.



**Value**

A list containing 1) the variance-covariance matrix of the estimators (assessed at the specific time points 'u' and 'v' such that  $u=v=\text{mean}(\text{data})$ ); 2) the variance of the mixture weight estimator; 3) the variance of the location shift estimator; 4) the variance of the unknown component cumulative distribution function at points 'u' and 'v' (useless for most of applications, explaining why 'u' and 'v' are set equal to  $\text{mean}(\text{data})$  by default, with no corresponding arguments here).

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
## Simulate data:
list.comp <- list(f = 'norm', g = 'norm')
list.param <- list(f = c(mean = 4, sd = 1), g = c(mean = 7, sd = 0.5))
sim.data <- rsimmix(n=140, unknownComp_weight=0.9, comp.dist=list.comp, comp.param=list.param)
## Estimate the location shift and mixture weight parameters in real-life setting:
list.comp <- list(f = NULL, g = 'norm')
list.param <- list(f = NULL, g = c(mean = 7, sd = 0.5))
estimators <- BVdk_estimParam(data = sim.data[['mixt.data']], method = "L-BFGS-B",
                             comp.dist = list.comp, comp.param = list.param)
## Estimate the variance of the two estimators (first mixture weight, then location shift):
BVdk_varCov_estimators(data = sim.data[['mixt.data']], loc = estimators[2], p = estimators[1],
                       comp.dist = list.comp, comp.param = list.param)
```

---

decontaminated_cdf	<i>Provide the decontaminated cumulative distribution function (CDF) of the unknown component in an admixture model</i>
--------------------	---

---

**Description**

Estimate the decontaminated CDF of the unknown component in the admixture model under study, after inversion of the admixture cumulative distribution function. Recall that an admixture model follows the cumulative distribution function (CDF)  $L$ , where  $L = p \cdot F + (1-p) \cdot G$ , with  $g$  a known CDF and  $p$  and  $f$  unknown quantities.

**Usage**

```
decontaminated_cdf(sample1, comp.dist, comp.param, estim.p)
```

**Arguments**

sample1            Observations of the sample under study.

comp.dist	A list with two elements corresponding to the component distributions (specified with R native names for these distributions) involved in the admixture model. The two elements refer to the unknown and known components of the admixture model. If there are unknown elements, they must be specified as 'NULL' objects (e.g. 'comp.dist' could be set to list(f1=NULL, g1='norm')).
comp.param	A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two elements refer to the parameters of unknown and known components of the admixture model. If there are unknown elements, they must be specified as 'NULL' objects (e.g. 'comp.param' could be set to list(f1=NULL, g1=list(mean=0,sd=1))).
estim.p	The estimated weight of the unknown component distribution, related to the proportion of the unknown component in the admixture model studied.

### Details

The decontaminated CDF is obtained by inverting the admixture CDF, given by  $L = p * F + (1-p) * G$ , to isolate the unknown component  $F$  after having estimated  $p$ . This means that  $F = (1/\hat{p}) * (\hat{L} - (1-p) * G)$ .

### Value

The decontaminated CDF  $F$  of the admixture model, as an of class 'stepfun' (step function).

### Author(s)

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

### Examples

```
##### Continuous support:
## Simulate data:
list.comp <- list(f1 = 'norm', g1 = 'norm',
                 f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 3, sd = 0.5), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=3500, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=3000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))

## Estimate the mixture weight in each of the sample in real-life setting:
list.comp <- list(f1 = NULL, g1 = 'norm',
                 f2 = NULL, g2 = 'norm')
list.param <- list(f1 = NULL, g1 = list(mean = 0, sd = 1),
                  f2 = NULL, g2 = list(mean = 5, sd = 2))
estimate <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], comp.dist = list.comp,
                          comp.param = list.param, with.correction = FALSE, n.integ = 1000)

## Determine the decontaminated version of the unknown CDF by inversion:
decontaminated_cdf(sample1 = sample1[['mixt.data']], comp.dist = list.comp[1:2],
                   comp.param = list.param[1:2], estim.p = estimate$prop.estim[1])
```

```
##### Countable discrete support:
list.comp <- list(f1 = 'pois', g1 = 'pois',
                 f2 = 'pois', g2 = 'pois')
list.param <- list(f1 = list(lambda = 3), g1 = list(lambda = 2),
                  f2 = list(lambda = 3), g2 = list(lambda = 4))
sample1 <- rsimmix(n=6000, unknownComp_weight=0.6, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=4500, unknownComp_weight=0.8, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))

## Estimate the mixture weight in each of the sample in real-life setting:
list.comp <- list(f1 = NULL, g1 = 'pois',
                 f2 = NULL, g2 = 'pois')
list.param <- list(f1 = NULL, g1 = list(lambda = 2),
                  f2 = NULL, g2 = list(lambda = 4))
estimate <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], comp.dist = list.comp,
                          comp.param = list.param, with.correction = FALSE, n.integ = 1000)
decontaminated_cdf(sample1 = sample1[['mixt.data']], comp.dist = list.comp[1:2],
                   comp.param = list.param[1:2], estim.p = estimate$prop.estim[1])

##### Finite discrete support:
list.comp <- list(f1 = 'multinom', g1 = 'multinom',
                 f2 = 'multinom', g2 = 'multinom')
list.param <- list(f1 = list(size=1, prob=c(0.3,0.4,0.3)), g1 = list(size=1, prob=c(0.6,0.3,0.1)),
                  f2 = list(size=1, prob=c(0.3,0.4,0.3)), g2 = list(size=1, prob=c(0.2,0.6,0.2)))
sample1 <- rsimmix(n=8000, unknownComp_weight=0.6, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=6000, unknownComp_weight=0.8, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))

list.comp <- list(f1 = NULL, g1 = 'multinom',
                 f2 = NULL, g2 = 'multinom')
list.param <- list(f1 = NULL, g1 = list(size=1, prob=c(0.6,0.3,0.1)),
                  f2 = NULL, g2 = list(size=1, prob=c(0.2,0.6,0.2)))
estimate <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], comp.dist = list.comp,
                          comp.param = list.param, with.correction = FALSE, n.integ = 1000)
decontaminated_cdf(sample1 = sample1[['mixt.data']], comp.dist = list.comp[1:2],
                   comp.param = list.param[1:2], estim.p = estimate$prop.estim[1])
```

---

decontaminated\_density

*Provide the decontaminated density of the unknown component in an admixture model.*

---

## Description

Estimate the decontaminated density of the unknown component in the admixture model under study, after inversion of the admixture cumulative distribution function. Recall that an admixture model follows the cumulative distribution function (CDF)  $L$ , where  $L = p * F + (1-p) * G$ , with  $g$  a known CDF and  $p$  and  $f$  unknown quantities.

**Usage**

```
decontaminated_density(sample1, comp.dist, comp.param, estim.p)
```

**Arguments**

sample1	Observations of the first sample under study.
comp.dist	A list with two elements corresponding to the component distributions (specified with R native names for these distributions) involved in the admixture model. The two elements refer to the unknown and known components of the admixture model. If there are unknown elements, they must be specified as 'NULL' objects (e.g. 'comp.dist' could be set to <code>list(f1=NULL, g1='norm')</code> ).
comp.param	A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two elements refer to the parameters of unknown and known components of the admixture model. If there are unknown elements, they must be specified as 'NULL' objects (e.g. 'comp.param' could be set to <code>list(f1=NULL, g1=list(mean=0,sd=1))</code> ).
estim.p	The estimated weight of the unknown component distribution, related to the proportion of the unknown component in the admixture model studied.

**Details**

The decontaminated density is obtained by inverting the admixture density, given by  $l = p*f + (1-p)*g$ , to isolate the unknown component  $f$  after having estimated  $p$ .

**Value**

A list containing the decontaminated density of the admixture model (of class 'function'), and the support of the observations (either discrete or continuous).

**Author(s)**

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**Examples**

```
## Simulate data:

##### Continuous support:
list.comp <- list(f1 = 'norm', g1 = 'norm',
                 f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 3, sd = 0.5), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=8000, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=7000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))

## Estimate the mixture weight in each of the sample in real-life setting:
list.comp <- list(f1 = NULL, g1 = 'norm',
```

```

      f2 = NULL, g2 = 'norm')
list.param <- list(f1 = NULL, g1 = list(mean = 0, sd = 1),
                  f2 = NULL, g2 = list(mean = 5, sd = 2))
estimate <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], comp.dist = list.comp,
                          comp.param = list.param, with.correction = FALSE, n.integ = 1000)
## Determine the decontaminated version of the unknown density by inversion:
decontaminated_density(sample1 = sample1[['mixt.data']], comp.dist = list.comp[1:2],
                       comp.param = list.param[1:2], estim.p = estimate$prop.estim[1])

##### Discrete support:
list.comp <- list(f1 = 'pois', g1 = 'pois',
                 f2 = 'pois', g2 = 'pois')
list.param <- list(f1 = list(lambda = 3), g1 = list(lambda = 2),
                  f2 = list(lambda = 3), g2 = list(lambda = 4))
sample1 <- rsimmix(n=7000, unknownComp_weight=0.6, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=6000, unknownComp_weight=0.8, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))

## Estimate the mixture weight in each of the sample in real-life setting:
list.comp <- list(f1 = NULL, g1 = 'pois',
                 f2 = NULL, g2 = 'pois')
list.param <- list(f1 = NULL, g1 = list(lambda = 2),
                  f2 = NULL, g2 = list(lambda = 4))
estimate <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], comp.dist = list.comp,
                          comp.param = list.param, with.correction = FALSE, n.integ = 1000)
## Determine the decontaminated version of the unknown density by inversion:
decontaminated_density(sample1 = sample1[['mixt.data']], comp.dist = list.comp[1:2],
                       comp.param = list.param[1:2], estim.p = estimate$prop.estim[1])

##### Finite discrete support:
list.comp <- list(f1 = 'multinom', g1 = 'multinom',
                 f2 = 'multinom', g2 = 'multinom')
list.param <- list(f1 = list(size=1, prob=c(0.3,0.4,0.3)), g1 = list(size=1, prob=c(0.6,0.3,0.1)),
                  f2 = list(size=1, prob=c(0.3,0.4,0.3)), g2 = list(size=1, prob=c(0.2,0.6,0.2)))
sample1 <- rsimmix(n=12000, unknownComp_weight=0.6, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=10000, unknownComp_weight=0.8, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))

## Estimate the mixture weight in each of the sample in real-life setting:
list.comp <- list(f1 = NULL, g1 = 'multinom',
                 f2 = NULL, g2 = 'multinom')
list.param <- list(f1 = NULL, g1 = list(size=1, prob=c(0.6,0.3,0.1)),
                  f2 = NULL, g2 = list(size=1, prob=c(0.2,0.6,0.2)))
estimate <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], comp.dist = list.comp,
                          comp.param = list.param, with.correction = FALSE, n.integ = 1000)
## Determine the decontaminated version of the unknown density by inversion:
decontaminated_density(sample1 = sample1[['mixt.data']], comp.dist = list.comp[1:2],
                       comp.param = list.param[1:2], estim.p = estimate$prop.estim[1])

```

**Description**

Given one or two sets of observations (two samples), the function provides with the most plausible type of support for the underlying random variables to be studied. Basically, if less than 3 percent of the observations have different values, we consider that the support is discrete. Otherwise, we consider it as a continuous support.

**Usage**

```
detect_support_type(sample1, sample2 = NULL)
```

**Arguments**

sample1            The first sample of observations under study.  
sample2            The second sample of observations under study.

**Value**

The type of support, either discrete or continuous.

**Author(s)**

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**Examples**

```
## Simulate the two mixture samples:
list.comp <- list(f1 = 'norm', g1 = 'norm',
                 f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 1, sd = 0.1), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=1500, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=2000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))
## Test the type of support:
detect_support_type(sample1[['mixt.data']], sample2[['mixt.data']])
```

---

estimVarCov\_empProcess

*Variance-covariance matrix of the empirical process in an admixture model*

---

**Description**

Estimate the variance-covariance matrix of some given empirical process, based on the Donsker correlation. Compute Donsker correlation between two time points (x,y) for some given empirical process with R code (another implementation in C++ is also available to speed up this computation).

**Usage**

```
estimVarCov_empProcess(
  x,
  y,
  obs.data,
  known.p = NULL,
  comp.dist = NULL,
  comp.param = NULL
)
```

**Arguments**

x	First time point considered for the computation of the correlation given the empirical process.
y	Second time point considered for the computation of the correlation given the same empirical process.
obs.data	Sample that permits to estimate the cumulative distribution function (cdf).
known.p	NULL by default (only useful to compute the exact Donsker correlation). The component weight dedicated to the unknown mixture component if available (in case of simulation studies)
comp.dist	NULL by default (only useful to compute the exact Donsker correlation). Otherwise, a list with two elements corresponding to component distributions (specified with R native names for these distributions) involved in the admixture model. All elements must be specified, for instance list(f='norm', g='norm').
comp.param	NULL by default (only useful to compute the exact Donsker correlation). Otherwise, a list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. All elements must be specified, for instance list(f=NULL, g=list(mean=0,sd=1)).

**Value**

The estimated variance-covariance matrix.

**Author(s)**

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**Examples**

```
## Simulate data:
list.comp <- list(f1 = 'norm', g1 = 'norm')
list.param <- list(f1 = list(mean = 12, sd = 0.4),
                  g1 = list(mean = 16, sd = 0.7))
obs.data <- rsimmix(n=2500, unknownComp_weight=0.5, comp.dist=list.comp, comp.param= list.param)
## Compute the variance-covariance matrix of the corresponding empirical process:
t <- seq(from = min(obs.data$mixt.data), to = max(obs.data$mixt.data), length = 50)
S2 <- sapply(t, function(s1) {
```

```

      sapply(t, function(s2) {
        estimVarCov_empProcess(x = s1, y = s2, obs.data = obs.data$mixt.data) })
    })
  lattice::wireframe(S2)

```

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gaussianity_test	<i>One-sample gaussianity test in admixture models using Bordes and Vandekerkhove estimation method</i>
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### Description

Perform the hypothesis test to know whether the unknown mixture component is gaussian or not, knowing that the known one has support on the real line ( $\mathbb{R}$ ). The case of non-gaussian known component can be overcome thanks to the basic transformation by cdf. Recall that an admixture model has probability density function (pdf)  $l = p*f + (1-p)*g$ , where  $g$  is the known pdf and  $l$  is observed (others are unknown). Requires optimization (to estimate the unknown parameters) as defined by Bordes & Vandekerkhove (2010), which means that the unknown mixture component must have a symmetric density.

### Usage

```

gaussianity_test(
  sample1,
  comp.dist,
  comp.param,
  K = 3,
  lambda = 0.2,
  conf.level = 0.95,
  support = c("Real", "Integer", "Positive", "Bounded.continuous")
)

```

### Arguments

sample1	Observed sample with mixture distribution given by $l = p*f + (1-p)*g$ , where $f$ and $p$ are unknown and $g$ is known.
comp.dist	List with two elements corresponding to the component distributions involved in the admixture model. Unknown elements must be specified as 'NULL' objects. For instance if 'f' is unknown: <code>list(f = NULL, g = 'norm')</code> .
comp.param	List with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R names for distributions. Unknown elements must be specified as 'NULL' objects (e.g. if 'f' is unknown: <code>list(f=NULL, g=list(mean=0,sd=1))</code> ).
K	Number of coefficients considered for the polynomial basis expansion.
lambda	Rate at which the normalization factor is set in the penalization rule for model selection (in $]0,1/2[$ ). See 'Details' below.



conf.level	The confidence level, default to 95 percent. Equals 1-alpha, where alpha is the level of the test (type-I error).
support	Support of the densities under consideration, useful to choose the polynomial orthonormal basis. One of 'Real', 'Integer', 'Positive', or 'Bounded.continuous'.

### Details

See the paper 'False Discovery Rate model Gaussianity test' (Pommeret & Vanderkerkhove, 2017).

### Value

A list of 6 elements, containing: 1) the rejection decision; 2) the p-value of the test; 3) the test statistic; 4) the variance-covariance matrix of the test statistic; 5) the selected rank for testing; and 6) a list of the estimates (unknown component weight 'p', shift location parameter 'mu' and standard deviation 's' of the symmetric unknown distribution).

### Author(s)

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### Examples

```
##### Under the null hypothesis H0.
## Parameters of the gaussian distribution to be tested:
list.comp <- list(f = "norm", g = "norm")
list.param <- list(f = c(mean = 2, sd = 0.5),
                  g = c(mean = 0, sd = 1))
## Simulate and plot the data at hand:
obs.data <- rsimmix(n = 150, unknownComp_weight = 0.9, comp.dist = list.comp,
                  comp.param = list.param)[['mixt.data']]
plot(density(obs.data))
## Performs the test:
list.comp <- list(f = NULL, g = "norm")
list.param <- list(f = NULL, g = c(mean = 0, sd = 1))
gaussianity_test(sample1 = obs.data, comp.dist = list.comp, comp.param = list.param,
                 K = 3, lambda = 0.1, conf.level = 0.95, support = 'Real')
```

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IBM_2samples_test	<i>Equality test of unknown component distributions in two admixture models with IBM approach</i>
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### Description

Two-sample test of the unknown component distribution in admixture models using Inversion - Best Matching (IBM) method. Recall that we have two admixture models with respective probability density functions (pdf)  $l_1 = p_1 f_1 + (1-p_1) g_1$  and  $l_2 = p_2 f_2 + (1-p_2) g_2$ , where  $g_1$  and  $g_2$  are known pdf and  $l_1$  and  $l_2$  are observed. Perform the following hypothesis test:  $H_0 : f_1 = f_2$  versus  $H_1 : f_1$  differs from  $f_2$ .

**Usage**

```

IBM_2samples_test(
  samples,
  known.p = NULL,
  comp.dist = NULL,
  comp.param = NULL,
  sim_U = NULL,
  n_sim_tab = 50,
  min_size = NULL,
  conf.level = 0.95,
  parallel = FALSE,
  n_cpu = 2
)

```

**Arguments**

<code>samples</code>	A list of the two observed samples, where each sample follows the mixture distribution given by $l = p*f + (1-p)*g$ , with $f$ and $p$ unknown and $g$ known.
<code>known.p</code>	(default to NULL) Numeric vector with two elements, the known (true) mixture weights.
<code>comp.dist</code>	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: <code>list(f1=NULL, g1='norm', f2=NULL, g2='norm')</code> .
<code>comp.param</code>	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: <code>: list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1))</code> .
<code>sim_U</code>	Random draws of the inner convergence part of the contrast as defined in the IBM approach (see 'Details' below).
<code>n_sim_tab</code>	Number of simulated gaussian processes used in the tabulation of the inner convergence distribution in the IBM approach.
<code>min_size</code>	(default to NULL, only used with 'ICV' testing method in the k-sample case, otherwise useless) Minimal size among all samples (needed to take into account the correction factor for the variance-covariance assessment).
<code>conf.level</code>	The confidence level of the 2-samples test, i.e. the quantile level to which the test statistic is compared.
<code>parallel</code>	(default to FALSE) Boolean to indicate whether parallel computations are performed (speed-up the tabulation).
<code>n_cpu</code>	(default to 2) Number of cores used when parallelizing.

**Details**

See the paper presenting the IBM approach at the following HAL weblink: <https://hal.science/hal-03201760>

**Value**

A list of five elements, containing : 1) the test statistic value; 2) the rejection decision; 3) the p-value of the test, 4) the estimated weights of the unknown component for each of the two admixture models, 5) the simulated distribution of the inner convergence regime (useful to perform the test when comparing to the extreme quantile of this distribution).

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**Examples**

```
##### Under the null hypothesis H0 :
## Simulate data:
list.comp <- list(f1 = "norm", g1 = "norm",
                 f2 = "norm", g2 = "norm")
list.param <- list(f1 = list(mean = 1, sd = 1), g1 = list(mean = 2, sd = 0.7),
                  f2 = list(mean = 1, sd = 1), g2 = list(mean = 3, sd = 1.2))
X.sim <- rsimmix(n= 1100, unknownComp_weight=0.85, comp.dist = list(list.comp$f1,list.comp$g1),
                comp.param = list(list.param$f1, list.param$g1))$mixt.data
Y.sim <- rsimmix(n= 1200, unknownComp_weight=0.75, comp.dist = list(list.comp$f2,list.comp$g2),
                comp.param = list(list.param$f2, list.param$g2))$mixt.data
list.comp <- list(f1 = NULL, g1 = "norm",
                 f2 = NULL, g2 = "norm")
list.param <- list(f1 = NULL, g1 = list(mean = 2, sd = 0.7),
                  f2 = NULL, g2 = list(mean = 3, sd = 1.2))
IBM_2samples_test(samples = list(X.sim, Y.sim), known.p = NULL, comp.dist = list.comp,
                  comp.param = list.param, sim_U = NULL, n_sim_tab = 6, min_size = NULL,
                  conf.level = 0.95, parallel = FALSE, n_cpu = 2)

##### Under the alternative H1 :
## Simulate data:
list.comp <- list(f1 = "norm", g1 = "norm",
                 f2 = "norm", g2 = "norm")
list.param <- list(f1 = list(mean = 1, sd = 1), g1 = list(mean = 2, sd = 0.7),
                  f2 = list(mean = 2, sd = 1), g2 = list(mean = 3, sd = 1.2))
X.sim <- rsimmix(n= 1100, unknownComp_weight=0.85, comp.dist = list(list.comp$f1,list.comp$g1),
                comp.param = list(list.param$f1, list.param$g1))$mixt.data
Y.sim <- rsimmix(n= 1200, unknownComp_weight=0.75, comp.dist = list(list.comp$f2,list.comp$g2),
                comp.param = list(list.param$f2, list.param$g2))$mixt.data
list.comp <- list(f1 = NULL, g1 = "norm",
                 f2 = NULL, g2 = "norm")
list.param <- list(f1 = NULL, g1 = list(mean = 2, sd = 0.7),
                  f2 = NULL, g2 = list(mean = 3, sd = 1.2))
IBM_2samples_test(samples = list(X.sim, Y.sim), known.p = NULL, comp.dist = list.comp,
```

```
comp.param = list.param, sim_U = NULL, n_sim_tab = 6, min_size = NULL,
conf.level = 0.95, parallel = FALSE, n_cpu = 2)
```

---

IBM\_empirical\_contrast

*Empirical computation of the contrast in the Inversion - Best Matching (IBM) method*

---

### Description

Defines the empirical version of the contrast in the IBM method, to be minimized in the optimization process. For further details about the contrast definition, see 'Details' below.

### Usage

```
IBM_empirical_contrast(
  par,
  fixed.p.X = NULL,
  sample1,
  sample2,
  G,
  comp.dist,
  comp.param
)
```

### Arguments

par	Numeric vector with two elements, corresponding to the two parameter values at which to compute the contrast. In practice the component weights for the two admixture models.
fixed.p.X	Arbitrary value chosen by the user for the component weight related to the unknown component of the first admixture model. Only useful for optimization when the known components of the two models are identical (G1=G2, leading to unidimensional optimization).
sample1	Observations of the first sample under study.
sample2	Observations of the second sample under study.
G	Distribution on which to integrate when calculating the contrast.
comp.dist	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: list(f1=NULL, g1='norm', f2=NULL, g2='norm').

`comp.param` A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: `: list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1))`.

## Details

See the paper presenting the IBM approach at the following HAL weblink: <https://hal.science/hal-03201760>

## Value

The empirical contrast value evaluated at parameter values.

## Author(s)

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## Examples

```
## Simulate data:
list.comp <- list(f1 = 'norm', g1 = 'norm',
                 f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 1, sd = 0.1), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=1500, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param = list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=2000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param = list(list.param$f2,list.param$g2))

## Create the distribution on which the contrast will be integrated:
G <- stats::rnorm(n = 1000, mean = sample(c(sample1[['mixt.data']], sample2[['mixt.data']]),
                                       size = 1000, replace = TRUE),
                 sd = density(c(sample1[['mixt.data']], sample2[['mixt.data']]))$bw)

## Compute the empirical contrast at parameters (p1,p2) = (0.2,0.7) in a real-life setting:
list.comp <- list(f1 = NULL, g1 = 'norm',
                 f2 = NULL, g2 = 'norm')
list.param <- list(f1 = NULL, g1 = list(mean = 0, sd = 1),
                  f2 = NULL, g2 = list(mean = 5, sd = 2))
IBM_empirical_contrast(par = c(0.2,0.7), fixed.p.X = NULL, sample1 = sample1[['mixt.data']],
                      sample2= sample2[['mixt.data']], G=G, comp.dist = list.comp, comp.param = list.param)
```

---

IBM_estimProp	<i>Estimate the weights related to the proportions of the unknown components of the two admixture models</i>
---------------	--

---

### Description

Estimate the component weights from the Inversion - Best Matching (IBM) method, related to the two admixture models with respective probability density function (pdf)  $l_1$  and  $l_2$ , such that:  $l_1 = p_1 * f_1 + (1-p_1)g_1$  and  $l_2 = p_2 f_2 + (1-p_2) * g_2$ , where  $g_1$  and  $g_2$  are the known component densities. For further details about IBM approach, see 'Details' below.

### Usage

```
IBM_estimProp(
  sample1,
  sample2,
  known.prop = NULL,
  comp.dist = NULL,
  comp.param = NULL,
  with.correction = TRUE,
  n.integ = 1000
)
```

### Arguments

sample1	Observations of the first sample under study.
sample2	Observations of the second sample under study.
known.prop	(optional) Numeric vector with two elements, respectively the component weight for the unknown component in the first and in the second samples.
comp.dist	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: <code>list(f1=NULL, g1='norm', f2=NULL, g2='norm')</code> .
comp.param	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: <code>: list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1))</code> .

`with.correction` Boolean indicating whether the solution (estimated proportions) should be adjusted or not (with the constant determined thanks to the exact proportion, usually unknown except in case of simulations).

`n.integ` Number of data points generated for the distribution on which to integrate.

### Details

See the paper presenting the IBM approach at the following HAL weblink: <https://hal.science/hal-03201760>

### Value

A list with the two estimates of the component weights for each of the admixture model, plus that of the theoretical model if specified.

### Author(s)

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### Examples

```
##### On a simulated example to see whether the true parameters are well estimated.
## Simulate data:
list.comp <- list(f1 = 'norm', g1 = 'norm',
                 f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 3, sd = 0.5), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=1500, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=2000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))
## Estimate the mixture weights of the two admixture models (provide hat(theta)_n and theta^c):
estim <- IBM_estimProp(sample1 = sample1[['mixt.data']], sample2 = sample2[['mixt.data']],
                      known.prop = c(0.5,0.7), comp.dist = list.comp, comp.param = list.param,
                      with.correction = FALSE, n.integ = 1000)
estim[['prop.estim']]
estim[['theo.prop.estim']]
##### On a real-life example (unknown component densities, unknown mixture weights).
list.comp <- list(f1 = NULL, g1 = 'norm',
                 f2 = NULL, g2 = 'norm')
list.param <- list(f1 = NULL, g1 = list(mean = 0, sd = 1),
                  f2 = NULL, g2 = list(mean = 5, sd = 2))
## Estimate the mixture weights of the two admixture models (provide only hat(theta)_n):
estim <- IBM_estimProp(sample1 = sample1[['mixt.data']], sample2 = sample2[['mixt.data']],
                      known.prop = NULL, comp.dist = list.comp, comp.param = list.param,
                      with.correction = FALSE, n.integ = 1000)
estim[['prop.estim']]
estim[['theo.prop.estim']]
```

---

 IBM\_estimVarCov\_gaussVect

*Nonparametric estimation of the variance-covariance matrix of the gaussian vector in IBM approach*

---

### Description

Estimate the variance-covariance matrix of the gaussian vector at point 'z', considering the use of Inversion - Best Matching (IBM) method to estimate the model parameters in two-sample admixture models. Recall that the two admixture models have respective probability density functions (pdf)  $l_1$  and  $l_2$ , such that:  $l_1 = p_1*f_1 + (1-p_1)g_1$  and  $l_2 = p_2*f_2 + (1-p_2)*g_2$ , where  $g_1$  and  $g_2$  are the known component densities. Further information for the IBM approach are given in 'Details' below.

### Usage

```
IBM_estimVarCov_gaussVect(
  x,
  y,
  estim.obj,
  fixed.p1 = NULL,
  known.p = NULL,
  sample1,
  sample2,
  min_size = NULL,
  comp.dist = NULL,
  comp.param = NULL
)
```

### Arguments

x	Time point at which the first (related to the first parameter) underlying empirical process is looked through.
y	Time point at which the second (related to the second parameter) underlying empirical process is looked through.
estim.obj	Object obtained from the estimation of the component weights related to the proportions of the unknown component in each of the two admixture models.
fixed.p1	Arbitrary value chosen by the user for the component weight related to the unknown component of the first admixture model. Only useful for optimization when the known components of the two models are identical ( $G_1=G_2$ , leading to unidimensional optimization).
known.p	(optional, NULL by default) Numeric vector with two elements, the known (true) mixture weights.
sample1	Observations of the first sample under study.
sample2	Observations of the second sample under study.



min_size	(optional, NULL by default) in the k-sample case, useful to provide the minimal size among all samples (needed to take into account the correction factor in variance-covariance assessment). Otherwise, useless.
comp.dist	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: <code>list(f1=NULL, g1='norm', f2=NULL, g2='norm')</code> .
comp.param	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: <code>: list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1))</code> .

### Details

See the paper presenting the IBM approach at the following HAL weblink: <https://hal.science/hal-03201760>

### Value

The estimated variance-covariance matrix of the gaussian vector  $Z = (\hat{p}_1, \hat{p}_2, D_n(z))$ , at location '(x,y)'.

### Author(s)

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### Examples

```
##### Analysis by simulated data:
## Simulate Gamma - Exponential admixtures :
list.comp <- list(f1 = "gamma", g1 = "exp",
                 f2 = "gamma", g2 = "exp")
list.param <- list(f1 = list(shape = 2, scale = 3), g1 = list(rate = 1/3),
                  f2 = list(shape = 2, scale = 3), g2 = list(rate = 1/5))
X.sim <- rsimmix(n=400, unknownComp_weight=0.8, comp.dist = list(list.comp$f1, list.comp$g1),
               comp.param = list(list.param$f1, list.param$g1))$mixt.data
Y.sim <- rsimmix(n=350, unknownComp_weight=0.9, comp.dist = list(list.comp$f2, list.comp$g2),
               comp.param = list(list.param$f2, list.param$g2))$mixt.data
## Real-life setting:
list.comp <- list(f1 = NULL, g1 = "exp",
                 f2 = NULL, g2 = "exp")
list.param <- list(f1 = NULL, g1 = list(rate = 1/3),
```

```

f2 = NULL, g2 = list(rate = 1/5))
## Estimate the unknown component weights in the two admixture models:
estim <- IBM_estimProp(sample1 = X.sim, sample2 = Y.sim, known.prop = NULL, comp.dist = list.comp,
  comp.param = list.param, with.correction = FALSE, n.integ = 1000)
IBM_estimVarCov_gaussVect(x = mean(X.sim), y = mean(Y.sim), estim.obj = estim,
  fixed.p1 = estim[["p.X.fixed"]], known.p = NULL, sample1 = X.sim,
  sample2 = Y.sim, min_size = NULL,
  comp.dist = list.comp, comp.param = list.param)

```

---

 IBM\_gap

*Difference between the unknown empirical cumulative distribution functions in two admixture models*

---

### Description

Compute the 'gap' between two unknown cumulative distribution functions (ecdf) at some given point, in admixture models with probability distribution function (pdf) given by  $l$  where  $l = p \cdot f + (1-p) \cdot g$ . Uses the inversion method to do so, i.e.  $f = (1/p) (l - (1-p) \cdot g)$ , where  $g$  represents the known component of the admixture model and  $p$  is the unknown proportion of the unknown component. Therefore, compute:  $D(z, L1, L2, p1, p2) = F1(z, L1, p1) - F2(z, L2, p2)$  This measure should be integrated over some domain to compute the global discrepancy, see further information in 'Details' below.

### Usage

```
IBM_gap(z, par, fixed.p1 = NULL, sample1, sample2, comp.dist, comp.param)
```

### Arguments

<code>z</code>	the point at which the difference between both unknown (estimated) component distributions is computed.
<code>par</code>	Numeric vector with two elements, corresponding to the weights of the unknown component for the two admixture models.
<code>fixed.p1</code>	(optional, NULL by default) Arbitrary value chosen by the user for the component weight related to the unknown component of the first admixture model. Only useful for optimization when the known components of the two models are identical ( $G1=G2$ , leading to unidimensional optimization).
<code>sample1</code>	Observations of the first sample under study.
<code>sample2</code>	Observations of the second sample under study.
<code>comp.dist</code>	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: <code>list(f1=NULL, g1='norm', f2=NULL, g2='norm')</code> .

`comp.param` A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: `: list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1))`.

### Details

See the paper presenting the IBM approach at the following HAL weblink: <https://hal.science/hal-03201760>

### Value

the gap evaluated at the specified point between the unknown components of the two observed samples.

### Author(s)

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

### Examples

```
list.comp <- list(f1 = 'norm', g1 = 'norm',
                f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 1, sd = 0.1), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=1500, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=2000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))
IBM_gap(z = 2.8, par = c(0.3,0.6), fixed.p1 = NULL, sample1 = sample1[['mixt.data']],
        sample2 = sample2[['mixt.data']], comp.dist = list.comp, comp.param = list.param)
```

---

IBM\_greenLight\_criterion

*Green-light criterion to decide whether to perform full equality test  
between unknown components between two admixture models*

---

### Description

Indicate whether there is need to perform the statistical test of equality between unknown components when comparing the unknown components of two samples following admixture models. Based on the IBM approach, see more in 'Details' below.

**Usage**

```

IBM_greenLight_criterion(
  estim.obj,
  sample1,
  sample2,
  comp.dist = NULL,
  comp.param = NULL,
  min_size = NULL,
  alpha = 0.05
)

```

**Arguments**

<code>estim.obj</code>	Object obtained from the estimation of the component weights related to the proportions of the unknown component in each of the two admixture models studied.
<code>sample1</code>	Observations of the first sample under study.
<code>sample2</code>	Observations of the second sample under study.
<code>comp.dist</code>	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: <code>list(f1=NULL, g1='norm', f2=NULL, g2='norm')</code> .
<code>comp.param</code>	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: <code>: list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1))</code> .
<code>min_size</code>	(optional, NULL by default) In the k-sample case, useful to provide the minimal size among all samples (needed to take into account the correction factor for variance-covariance assessment). Otherwise, useless.
<code>alpha</code>	Confidence level at which the criterion is assessed (used to compute the confidence bands of the estimators of the unknown component weights).

**Details**

See the paper presenting the IBM approach at the following HAL weblink: <https://hal.science/hal-03201760>

**Value**

A boolean indicating whether it is useful or useless to tabulate the contrast distribution in order to answer the testing problem ( $f1 = f2$ ).

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
## Simulate data:
list.comp <- list(f1 = 'norm', g1 = 'norm',
                 f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 3, sd = 0.5), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=550, unknownComp_weight=0.7, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=450, unknownComp_weight=0.8, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))

## Estimate the unknown component weights in the two admixture models in real-life setting:
list.comp <- list(f1 = NULL, g1 = 'norm',
                 f2 = NULL, g2 = 'norm')
list.param <- list(f1 = NULL, g1 = list(mean = 0, sd = 1),
                  f2 = NULL, g2 = list(mean = 5, sd = 2))
estim <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], known.prop = NULL,
                      comp.dist = list.comp, comp.param = list.param,
                      with.correction = FALSE, n.integ = 1000)
IBM_greenLight_criterion(estim.obj = estim, sample1 = sample1[['mixt.data']],
                         sample2 = sample2[['mixt.data']], comp.dist = list.comp,
                         comp.param = list.param, min_size = NULL, alpha = 0.05)
```

---

IBM\_hessian\_contrast *Hessian matrix of the contrast function in the Inversion - Best Matching (IBM) method*

---

**Description**

Compute the hessian matrix of the contrast as defined in the IBM approach, at point  $(p_1, p_2)$ . Here, based on two samples following admixture models, where we recall that admixture models have probability distribution function (pdf) given by  $l$  where  $l = p*f + (1-p)*g$ , where  $g$  represents the only known quantity and  $l$  is the pdf of the observed sample. See 'Details' below for further information about the definition of the contrast.

**Usage**

```
IBM_hessian_contrast(
  par,
  fixed.p1 = NULL,
  known.p = NULL,
  sample1,
  sample2,
```

```

    G,
    comp.dist = NULL,
    comp.param = NULL
  )

```

### Arguments

par	Numeric vector with two elements (corresponding to the two unknown component weights) at which the hessian is computed.
fixed.p1	(optional, NULL by default) Arbitrary value chosen by the user for the component weight related to the unknown component of the first admixture model. Only useful for optimization when the known components of the two models are identical (G1=G2, leading to unidimensional optimization).
known.p	(optional, NULL by default) Numeric vector with two elements, the known (true) mixture weights.
sample1	Observations of the first sample under study.
sample2	Observations of the second sample under study.
G	Distribution on which to integrate when calculating the contrast.
comp.dist	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: list(f1=NULL, g1='norm', f2=NULL, g2='norm').
comp.param	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: : list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1)).

### Details

See the paper presenting the IBM approach at the following HAL weblink: <https://hal.science/hal-03201760>

### Value

the hessian matrix of the contrast.

### Author(s)

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
## Simulate data:
list.comp <- list(f1 = 'norm', g1 = 'norm',
                 f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 1, sd = 0.1), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=1500, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=2000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))

## Define the distribution over which to integrate:
fit.all <- stats::density(x = c(sample1[['mixt.data']],sample2[['mixt.data']]))
G <- stats::rnorm(n = 1000, mean = sample(c(sample1[['mixt.data']], sample2[['mixt.data']])),
                 size = 1000, replace = TRUE), sd = fit.all$bw)

## Evaluate the hessian matrix at point (p1,p2) = (0.3,0.6):
list.comp <- list(f1 = NULL, g1 = 'norm',
                 f2 = NULL, g2 = 'norm')
list.param <- list(f1 = NULL, g1 = list(mean = 0, sd = 1),
                  f2 = NULL, g2 = list(mean = 5, sd = 2))
IBM_hessian_contrast(par = c(0.3,0.6), fixed.p1 = NULL, known.p = NULL,
                    sample1 = sample1[['mixt.data']], sample2 = sample2[['mixt.data']], G = G,
                    comp.dist = list.comp, comp.param = list.param)
```

---

IBM_k_samples_test	<i>Equality test of unknown component distributions in K admixture models, with IBM approach</i>
--------------------	--

---

**Description**

Test hypothesis on the unknown component of K ( $K > 1$ ) admixture models using Inversion - Best Matching method. K-samples test of the unknown component distribution in admixture models using Inversion - Best Matching (IBM) method. Recall that we have K populations following admixture models, each one with probability density functions (pdf)  $l_k = p_k * f_k + (1-p_k) * g_k$ , where  $g_k$  is the known pdf and  $l_k$  corresponds to the observed sample. Perform the following hypothesis test:  $H_0 : f_1 = \dots = f_K$  against  $H_1 : f_i$  differs from  $f_j$  (i different from j, and  $i, j$  in  $1, \dots, K$ ).

**Usage**

```
IBM_k_samples_test(
  samples = NULL,
  sim_U = NULL,
  n_sim_tab = 100,
  comp.dist = NULL,
  comp.param = NULL,
  conf.level = 0.95,
  tune.penalty = TRUE,
```

```

parallel = FALSE,
n_cpu = 2
)

```

### Arguments

<code>samples</code>	A list of the K samples to be studied, all following admixture distributions.
<code>sim_u</code>	(default to NULL) Random draws of the inner convergence part of the contrast as defined in the IBM approach (see 'Details' below).
<code>n_sim_tab</code>	Number of simulated gaussian processes when tabulating the inner convergence distribution in the IBM approach.
<code>comp.dist</code>	A list with 2*K elements corresponding to the component distributions (specified with R native names for these distributions) involved in the K admixture models. Elements, grouped by 2, refer to the unknown and known components of each admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows with K = 3: <code>list(f1 = NULL, g1 = 'norm', f2 = NULL, g2 = 'norm', f3 = NULL, g3 = 'norm')</code> .
<code>comp.param</code>	A list with 2*K elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Elements, grouped by 2, refer to the parameters of unknown and known components of each admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows (with K = 3): <code>list(f1 = NULL, g1 = list(mean=0,sd=1), f2 = NULL, g2 = list(mean=3,sd=1.1), f3 = NULL, g3 = list(mean=-2,sd=0.6))</code> .
<code>conf.level</code>	The confidence level of the K-sample test.
<code>tune.penalty</code>	A boolean that allows to choose between a classical penalty term or an optimized penalty embedding some tuning parameters (automatically optimized). Optimized penalty is particularly useful for low sample size.
<code>parallel</code>	(default to FALSE) Boolean indicating whether parallel computations are performed.
<code>n_cpu</code>	(default to 2) Number of cores used when parallelizing.

### Details

See the paper at the following HAL weblink: <https://hal.science/hal-04129130>

### Value

A list of ten elements, containing: 1) the rejection decision; 2) the test p-value; 3) the terms involved in the test statistic; 4) the test statistic value; 5) the selected rank (number of terms involved in the test statistic); 6) the value of the penalized test statistic; 7) a boolean indicating whether the applied penalty rule is that under the null H0; 8) the sorted contrast values; 9) the 95th-quantile of the contrast distribution; 10) the final terms of the statistic; and 11) the contrast matrix.



**Author(s)**

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**Examples**

```
##### Under the null hypothesis H0 (with K=3 populations):
## Specify the parameters of the mixture models for simulation:
list.comp <- list(f1 = "norm", g1 = "norm",
                 f2 = "norm", g2 = "norm",
                 f3 = "norm", g3 = "norm")
list.param <- list(f1 = list(mean = 0, sd = 1), g1 = list(mean = 2, sd = 0.7),
                  f2 = list(mean = 0, sd = 1), g2 = list(mean = 4, sd = 1.1),
                  f3 = list(mean = 0, sd = 1), g3 = list(mean = -3, sd = 0.8))
## Simulate the data:
sim1 <- rsimmix(n = 1000, unknownComp_weight = 0.8, comp.dist = list(list.comp$f1, list.comp$g1),
               comp.param = list(list.param$f1, list.param$g1))$mixt.data
sim2 <- rsimmix(n = 1300, unknownComp_weight = 0.6, comp.dist = list(list.comp$f2, list.comp$g2),
               comp.param = list(list.param$f2, list.param$g2))$mixt.data
sim3 <- rsimmix(n = 1100, unknownComp_weight = 0.7, comp.dist = list(list.comp$f3, list.comp$g3),
               comp.param = list(list.param$f3, list.param$g3))$mixt.data
## Back to the context of admixture models, where one mixture component is unknown:
list.comp <- list(f1 = NULL, g1 = "norm",
                 f2 = NULL, g2 = "norm",
                 f3 = NULL, g3 = "norm")
list.param <- list(f1 = NULL, g1 = list(mean = 2, sd = 0.7),
                  f2 = NULL, g2 = list(mean = 4, sd = 1.1),
                  f3 = NULL, g3 = list(mean = -3, sd = 0.8))
## Perform the 3-samples test:
IBM_k_samples_test(samples = list(sim1, sim2, sim3), sim_U= NULL, n_sim_tab = 20,
                  comp.dist = list.comp, comp.param = list.param, conf.level = 0.95,
                  tune.penalty = FALSE, parallel = FALSE, n_cpu = 2)

##### Now under the alternative H1:
list.comp <- list(f1 = "norm", g1 = "norm",
                 f2 = "norm", g2 = "norm",
                 f3 = "norm", g3 = "norm")
list.param <- list(f1 = list(mean = 0, sd = 1), g1 = list(mean = 2, sd = 0.7),
                  f2 = list(mean = 0, sd = 1), g2 = list(mean = 4, sd = 1.1),
                  f3 = list(mean = 2, sd = 0.7), g3 = list(mean = 3, sd = 0.8))
sim1 <- rsimmix(n = 3000, unknownComp_weight = 0.8, comp.dist = list(list.comp$f1, list.comp$g1),
               comp.param = list(list.param$f1, list.param$g1))$mixt.data
sim2 <- rsimmix(n = 3300, unknownComp_weight = 0.6, comp.dist = list(list.comp$f2, list.comp$g2),
               comp.param = list(list.param$f2, list.param$g2))$mixt.data
sim3 <- rsimmix(n = 3100, unknownComp_weight = 0.7, comp.dist = list(list.comp$f3, list.comp$g3),
               comp.param = list(list.param$f3, list.param$g3))$mixt.data
list.comp <- list(f1 = NULL, g1 = "norm",
                 f2 = NULL, g2 = "norm",
                 f3 = NULL, g3 = "norm")
list.param <- list(f1 = NULL, g1 = list(mean = 2, sd = 0.7),
                  f2 = NULL, g2 = list(mean = 4, sd = 1.1),
                  f3 = NULL, g3 = list(mean = 3, sd = 0.8))
```

```
IBM_k_samples_test(samples = list(sim1,sim2,sim3), sim_U= NULL, n_sim_tab = 20,
                  comp.dist = list.comp, comp.param = list.param, conf.level = 0.95,
                  tune.penalty = FALSE, parallel = FALSE, n_cpu = 2)
```

---

IBM\_tabul\_stochasticInteg

*Distribution of the contrast in the Inversion - Best Matching (IBM) method*

---

## Description

Tabulate the distribution related to the inner convergence part of the contrast, by simulating trajectories of gaussian processes and applying some transformations. Useful to perform the test hypothesis then, by retrieving the (1-alpha)-quantile of interest. See 'Details' below and the cited paper therein for further information.

## Usage

```
IBM_tabul_stochasticInteg(
  n.sim = 200,
  n.varCovMat = 100,
  sample1 = NULL,
  sample2 = NULL,
  min_size = NULL,
  comp.dist = NULL,
  comp.param = NULL,
  parallel = FALSE,
  n_cpu = 2
)
```

## Arguments

n.sim	Number of trajectories of simulated gaussian processes (number of random draws for tabulation).
n.varCovMat	Number of time points on which gaussian processes are simulated.
sample1	Observations of the first sample under study.
sample2	Observations of the second sample under study.
min_size	(default to NULL) In the k-sample case, useful to provide the minimal size among all samples. Otherwise, useless.
comp.dist	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: list(f1=NULL, g1='norm', f2=NULL, g2='norm').



---

 IBM\_theoretical\_contrast

*Theoretical contrast in the Inversion - Best Matching (IBM) method*


---

## Description

Defines the theoretical contrast in the IBM approach. Useful in case of simulation studies, since all parameters are known to the user. For further information about the considered contrast in the IBM approach, see 'Details' below.

## Usage

```
IBM_theoretical_contrast(
  par,
  theo.par,
  fixed.p.X = NULL,
  G = NULL,
  comp.dist,
  comp.param,
  sample1,
  sample2
)
```

## Arguments

par	Numeric vector with two elements, corresponding to the two parameter values at which to compute the contrast. In practice the component weights for the two admixture models.
theo.par	Numeric vector with two elements, the known (true) mixture weights.
fixed.p.X	Arbitrary value chosen by the user for the component weight related to the unknown component of the first admixture model. Only useful for optimization when the known components of the two models are identical ( $G_1=G_2$ , leading to unidimensional optimization).
G	Distribution on which to integrate when calculating the contrast.
comp.dist	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. No unknown elements permitted. For instance, 'comp.dist' could be specified as follows: <code>list(f1='rnorm', g1='norm', f2='rnorm', g2='norm')</code> .
comp.param	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture

model. No unknown elements permitted. For instance, 'comp.param' could be specified as follows: : list(f1 = list(mean=2,sd=0.3), g1 = list(mean=0,sd=1), f2 = list(mean=2,sd=0.3), g2 = list(mean=3,sd=1.1)).

sample1      Observations of the first sample under study.  
 sample2      Observations of the second sample under study.

### Details

See the paper presenting the IBM approach at the following HAL weblink: <https://hal.science/hal-03201760>

### Value

The theoretical contrast value evaluated at parameter values.

### Author(s)

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

### Examples

```
## Simulate data:
list.comp <- list(f1 = 'norm', g1 = 'norm',
                 f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 1, sd = 0.1), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=1500, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=2000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param=list(list.param$f2,list.param$g2))
## Create the distribution on which the contrast will be integrated:
G <- stats::rnorm(n = 1000, mean = sample(c(sample1[['mixt.data']],sample2[['mixt.data']])),
                 size = 1000, replace = TRUE),
      sd = stats::density(c(sample1[['mixt.data']],sample2[['mixt.data']]))$bw)
## Compute the theoretical contrast at parameters (p1,p2) = (0.2,0.7):
IBM_theoretical_contrast(par = c(0.2,0.7), theo.par = c(0.5,0.7), fixed.p.X = NULL, G = G,
                        comp.dist = list.comp, comp.param = list.param,
                        sample1 = sample1[['mixt.data']], sample2 = sample2[['mixt.data']])
```

**Description**

Compute the gap between the unknown cumulative distribution functions of the two considered admixture models at some given point, where each admixture model has probability distribution function (pdf) given by  $l$  where  $l = p*f + (1-p)*g$ . Uses the inversion method to do so, i.e.  $f = (1/p) * (l - (1-p)g)$ , where  $g$  represents the known component of the admixture model and  $p$  is the proportion of the unknown component. This difference must be integrated over some domain to compute the global discrepancy, as introduced in the paper presenting the IBM approach (see 'Details' below).

**Usage**

```
IBM_theoretical_gap(z, par, known.p = c(0.5, 0.5), comp.dist, comp.param)
```

**Arguments**

<code>z</code>	Point at which the difference between the unknown component distributions of the two considered admixture models is computed.
<code>par</code>	Numeric vector with two elements, corresponding to the two parameter values at which to compute the gap. In practice the component weights for the two admixture models.
<code>known.p</code>	Numeric vector with two elements, the known (true) mixture weights.
<code>comp.dist</code>	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. No unknown elements permitted. For instance, 'comp.dist' could be specified as follows: <code>list(f1='rnorm', g1='norm', f2='rnorm', g2='norm')</code> .
<code>comp.param</code>	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. No unknown elements permitted. For instance, 'comp.param' could be specified as follows: <code>: list(f1 = list(mean=2,sd=0.3), g1 = list(mean=0,sd=1), f2 = list(mean=2,sd=0.3), g2 = list(mean=3,sd=1.1))</code> .

**Details**

See the paper presenting the IBM approach at the following HAL weblink: <https://hal.science/hal-03201760>

**Value**

The gap between F1 and F2 (unknown components of the two admixture models), evaluated at the specified point.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
list.comp <- list(f1 = 'norm', g1 = 'norm',
                f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 1, sd = 0.1), g2 = list(mean = 5, sd = 2))
IBM_theoretical_gap(z = 2.8, par = c(0.3,0.6), known.p = c(0.5,0.5),
                   comp.dist = list.comp, comp.param = list.param)
```

---

is_equal_knownComp	<i>Test for equality of the known components between two admixture models</i>
--------------------	---

---

**Description**

Test if the known components coming from the two two-components admixture models are the same.

**Usage**

```
is_equal_knownComp(comp.dist, comp.param)
```

**Arguments**

comp.dist	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: list(f1=NULL, g1='norm', f2=NULL, g2='norm').
comp.param	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: : list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1)).

**Value**

A boolean (TRUE if the known components are the same, otherwise FALSE).

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
list.comp <- list(f1 = 'norm', g1 = 'norm',
                f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 2, sd = 0.3), g2 = list(mean = 0, sd = 1))
is_equal_knownComp(comp.dist = list.comp, comp.param = list.param)
```

---

kernel\_cdf

*Kernel estimation*

---

**Description**

Functions to perform the estimation of cumulative distribution function (cdf) by kernel estimators (with a non-gaussian kernel).

**Usage**

```
kernel_cdf(u, h)
```

**Arguments**

u                    the point at which the estimation is made.  
h                    window of the kernel estimation.

**Value**

the estimated value of the cdf.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
kernel_cdf(0.4, 0.5)
```



---

kernel_density	<i>Kernel estimation</i>
----------------	--------------------------

---

**Description**

Functions to perform the estimation of probability density function (pdf) by kernel estimators (with a non-gaussian kernel).

**Usage**

```
kernel_density(u, h)
```

**Arguments**

u	the point at which the estimation is made.
h	window of the kernel estimation.

**Value**

the estimated value of the pdf.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
kernel_density(0.4,0.5)
```

---

knownComp_to_uniform	<i>Transforms the known component of the admixture distribution to a Uniform distribution</i>
----------------------	---

---

**Description**

In admixture such that the probability density function (pdf) follows  $l = p*f + (1-p)*g$ , where  $p$  is the unknown weight and  $f$  is the unknown component distribution: transforms  $g$  of the two-component mixture distribution to a Uniform distribution. Useful to use Patra and Sen estimator for the estimation of the unknown weight  $p$ .

**Usage**

```
knownComp_to_uniform(data, comp.dist, comp.param)
```

**Arguments**

data	Observations of the sample under study, following an admixture distribution.
comp.dist	A list with two elements corresponding to component distributions (specified with R native names for these distributions) involved in the admixture model. Unknown elements must be specified as 'NULL' objects, e.g. when 'f' is unknown: <code>list(f=NULL, g='norm')</code> .
comp.param	A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Unknown elements must be specified as 'NULL' objects, e.g. if 'f' is unknown: <code>list(f=NULL, g=list(mean=0,sd=1))</code> .

**Value**

The transformed data, i.e. the transformed mixture distribution where the known component `g` now follows a `Uniform(0,1)` distribution.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
## Simulate data:
list.comp <- list(f1 = 'norm', g1 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5),
                  g1 = list(mean = 0, sd = 1))
sample1 <- rsimmix(n=1500, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param=list(list.param$f1,list.param$g1))
plot_mixt_density(samples = list(sample1[['mixt.data']]), support = 'continuous')
## Transform the known component into a Uniform(0,1) distribution:
list.comp <- list(f1 = NULL, g1 = 'norm')
list.param <- list(f1 = NULL, g1 = list(mean = 0, sd = 1))
transformed_data <- knownComp_to_uniform(data = sample1[['mixt.data']],
                                       comp.dist = list.comp, comp.param = list.param)
plot_mixt_density(samples = list(transformed_data), support = 'continuous')
```

---

milkyWay

*Heliocentric velocity measured for the Milky Way (from Walker, M. G., M. Mateo, E. W. Olszewski, O. Y. Gnedin, X. Wang, B. Sen, and M. Woodroffe (2007). Velocity dispersion profiles of seven dwarf spheroidal galaxies. Astrophysical J. 667(1), L53–L56).*

---

**Description**

Heliocentric velocity measured for the Milky Way (from Walker, M. G., M. Mateo, E. W. Olszewski, O. Y. Gnedin, X. Wang, B. Sen, and M. Woodroffe (2007). Velocity dispersion profiles of seven dwarf spheroidal galaxies. Astrophysical J. 667(1), L53–L56).

**Usage**

```
milkyWay
```

**Format**

A data frame with 170,601 rows and 1 column:

**V1** Heliocentric velocity measurements of the Milky way

**Source**

[https://www.aanda.org/articles/aa/full\\_html/2018/08/aa32905-18/aa32905-18.html](https://www.aanda.org/articles/aa/full_html/2018/08/aa32905-18/aa32905-18.html)

---

mortality_sample	<i>Dataset giving exposure-to-death (population size) and number of deaths for males in eleven european countries, with ages ranging from 30 years old to 85 years old.</i>
------------------	---

---

**Description**

Dataset giving exposure-to-death (population size) and number of deaths for males in eleven european countries, with ages ranging from 30 years old to 85 years old.

**Usage**

```
mortality_sample
```

**Format**

Two different lists related to the reduced (subsample) population size and reduced number of deaths in eleven european countries, for male people aged 30 years old to 85 years old between 1908 and 2020. The data were exported from the Human Mortality Database (HMD).

An evolving data frame of exposure-to-death and number of deaths in Belgium, Switzerland, Denmark, Spain, Finland, France, United Kingdom, Italia, The Netherlands, Norway and Sweden.

**XP** A list of eleven elements (one for each country) giving a subset of the exposure-to-death (or reduced population size), each element having 56 rows (ages 30-85) and 113 columns (period 1908-2020)

**DX** A list of eleven elements (one for each country) giving a subset of the number of deaths, each element having 56 rows (ages 30-85) and 113 columns (period 1908-2020)

**names** A list of eleven elements giving the names of the countries, in the same order as the elements in other lists

**Source**

<https://www.mortality.org>

---

orthoBasis_coef	<i>Compute expansion coefficients in a given orthonormal polynomial basis.</i>
-----------------	--

---

### Description

Compute the coefficients corresponding to the decomposition of some density in a given orthonormal polynomial basis.

### Usage

```
orthoBasis_coef(
  data,
  comp.dist = NULL,
  comp.param = NULL,
  supp = c("Real", "Integer", "Positive", "Bounded.continuous"),
  degree,
  m = 3,
  other = NULL
)
```

### Arguments

data	Observed sample from which the coefficients are calculated. Can be NULL if 'comp.dist' and 'comp.param' are specified.
comp.dist	(default to NULL) A list with two elements corresponding to component distributions (specified with R native names for these distributions) involved in the admixture model. Unknown elements must be specified as 'NULL' objects (for instance unknown 'f': list(f=NULL, g='norm')).
comp.param	(default to NULL) A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. Unknown elements must be specified as 'NULL' objects. For instance if 'f' is unknown: list(f = NULL, g = list(mean=0,sd=1)).
supp	Support of the density considered.
degree	Degree up to which the polynomial basis is built.
m	(default to 3) Only used when support is 'Integer'. Corresponds to the mean of the reference measure, i.e. Poisson(m).
other	(default to NULL) A list to precise bounds when the support is bounded, where the second and fourth elements give bounds.

### Value

The list composed of 'degree' elements, each element being a numeric vector (with sample size) where each value represents the k-th order coefficient found when decomposing the density in the orthonormal polynomial basis.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
## Simulate data:
sample1 <- rnorm(n = 7000, mean = 3, sd = 1)
## Compute the expansion coefficients in the orthonormal polynomial basis:
coeff <- orthoBasis_coef(data = sample1, comp.dist = NULL, comp.param = NULL, supp = 'Real',
                        degree = 3, m = 3, other = NULL)

sapply(coeff, mean)
## No observed data and decomposition of the known component of the admixture model:
coeff <- orthoBasis_coef(data = NULL, comp.dist = list(NULL, 'norm'),
                        comp.param=list(NULL,list(mean=3,sd=1)), supp = 'Real', degree=3, m=3, other = NULL)
sapply(coeff, mean)
```

---

orthoBasis_test_H0	<i>Equality test of unknown components between two admixture models using polynomial basis expansions</i>
--------------------	---

---

**Description**

Test the null hypothesis ( $H_0: f_1=f_2$ ) using the decomposition of unknown densities of the two admixture distributions in an adequate orthonormal polynomial basis. Recall that we have two admixture models with respective probability density functions (pdf)  $l_1 = p_1*f_1 + (1-p_1)g_1$  and  $l_2 = p_2*f_2 + (1-p_2)*g_2$ , where  $g_1$  and  $g_2$  are the only known elements. The admixture weights  $p_1$  and  $p_2$  thus have to be estimated. For further information on this method, see 'Details' below.

**Usage**

```
orthoBasis_test_H0(
  samples,
  known.p = NULL,
  comp.dist = NULL,
  comp.param = NULL,
  known.coef = NULL,
  K = 3,
  nb.ssEch = 2,
  s = 0.49,
  var.explicit = TRUE,
  nb.echBoot = NULL,
  support = c("Real", "Integer", "Positive", "Bounded.continuous", "Bounded.discrete"),
  bounds.supp = NULL,
  est.method = c("BVdk", "PS"),
  uniformized.knownComp_data = NULL
)
```

**Arguments**

<code>samples</code>	A list of the two observed samples, where each sample follows the mixture distribution given by $l = p*f + (1-p)*g$ , with $f$ and $p$ unknown and $g$ known.
<code>known.p</code>	(default to NULL) Numeric vector with two elements, respectively the component weight for the unknown component in the first and in the second samples.
<code>comp.dist</code>	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: <code>list(f1=NULL, g1='norm', f2=NULL, g2='norm')</code> .
<code>comp.param</code>	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: <code>: list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1))</code> .
<code>known.coef</code>	Coefficients in the polynomial basis expansion, corresponding to the known component densities $g1$ and $g2$ .
<code>K</code>	Number of coefficients considered for the polynomial basis expansion.
<code>nb.ssEch</code>	Number of subsamples created from the original data to decorrelate the estimation of the different parameters.
<code>s</code>	Rate at which the normalization factor is set in the penalization rule for model selection (in $]0,1/2[$ ), see 'Details'.
<code>var.explicit</code>	Boolean that allows to choose between explicit assessment of the variance of the test statistic or not (FALSE=bootstrap), FIXME : it seems that bootstrap procedure does not work in the context of admixtures.
<code>nb.echBoot</code>	number of bootstrap samples if 'var.explicit' is set to FALSE.
<code>support</code>	support of the densities under consideration, useful to choose the polynomial orthonormal basis.
<code>bounds.supp</code>	(default to NULL) useful if <code>support = 'bounded'</code> , a list of minimum and maximum bounds, specified as following: <code>list( list(min.f1,min.g1,min.f2,min.g2) , list(max.f1,max.g1,max.f2,max.g2) )</code>
<code>est.method</code>	Estimation method to get the component weights, either 'PS' (Patra and Sen estimation) or 'BVdk' (Bordes and Vendekerkhove estimation).
<code>uniformized.knownComp_data</code>	(default to NULL) Only useful if 'est.method' has been set to 'PS', and for real-life applications where the distribution of the known component of the admixture model is also unknown. In this case, this known component is previously made uniformly(0,1)-distributed by applying the empirical cumulative distribution of the known component function on the data. This means that all 'comp.dist' and 'comp.param' must be set to NULL.

**Details**

See the paper on HAL website: <https://hal.science/hal-03692868>

**Value**

A list with six elements containing: 1) the rejection decision; 2) the p-value of the test; 3) the test statistic; 4) the variance-covariance matrix of the test statistic; 5) selected rank for testing, and 6) estimates of the two component weights.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
##### Using Bordes and Vandekerkhove estimation (valid if symmetric unknown component densities).
#### Under the null hypothesis H0.
## Simulate data:
list.comp <- list(f1 = "norm", g1 = "norm",
                 f2 = "norm", g2 = "norm")
list.param <- list(f1 = c(mean = 1, sd = 1), g1 = c(mean = 4, sd = 1),
                  f2 = c(mean = 1, sd = 1), g2 = c(mean = 5, sd = 0.5))
sim.X <- rsimmix(n = 250, unknownComp_weight=0.9, comp.dist = list(list.comp$f1, list.comp$g1),
                 comp.param = list(list.param$f1, list.param$g1))$mixt.data
sim.Y <- rsimmix(n = 300, unknownComp_weight=0.8, comp.dist = list(list.comp$f2, list.comp$g2),
                 comp.param = list(list.param$f2, list.param$g2))$mixt.data
plot_mixt_density(samples = list(sim.X, sim.Y), support = "continuous")
## Perform the hypothesis test in real-life conditions:
list.comp <- list(f1 = NULL, g1 = "norm",
                 f2 = NULL, g2 = "norm")
list.param <- list(f1 = NULL, g1 = c(mean = 4, sd = 1),
                  f2 = NULL, g2 = c(mean = 5, sd = 0.5))
test <- orthoBasis_test_H0(samples = list(sim.X, sim.Y),
                           known.p=NULL, comp.dist = list.comp, comp.param = list.param, known.coef=NULL, K=3,
                           nb.ssEch = 2, s = 0.25, var.explicit=TRUE, nb.echBoot=NULL, support = 'Real',
                           bounds.supp = NULL, est.method = 'BVdk', uniformized.knownComp_data = NULL)
test$rejection_rule
```

**Description**

Estimation of unknown elements (by Patra and Sen method) under the admixture model with probability density function  $l = p*f + (1-p)*g$ , where  $g$  is the known component of the two-component admixture,  $p$  is the unknown proportion of the unknown component distribution  $f$ . The estimated unknown component weight  $p$  is selected using a cross-validation technique that helps to choose the right penalization, see 'Details' below for further information.

**Usage**

```
PatraSen_cv_mixmodel(
  data,
  folds = 10,
  reps = 1,
  cn.s = NULL,
  cn.length = NULL,
  gridsize = 200
)
```

**Arguments**

<code>data</code>	Sample where the known component density of the admixture model has been transformed into a Uniform(0,1) distribution.
<code>folds</code>	(default to 10) Number of folds used for cross-validation.
<code>reps</code>	(default to 1) Number of replications for cross-validation.
<code>cn.s</code>	(default to NULL) A sequence of 'c.n' to be used for cross-validation (vector of values).
<code>cn.length</code>	(default to NULL) Number of equally spaced tuning parameter (between $.001 \times \log(\log(n))$ and $0.2 \times \log(\log(n))$ ). Values to search from.
<code>gridsize</code>	(default to 200) Number of equally spaced points (between 0 and 1) to evaluate the distance function. Larger values are more computationally intensive but also lead to more accurate estimates.

**Details**

See Patra, R.K. and Sen, B. (2016); Estimation of a Two-component Mixture Model with Applications to Multiple Testing; JRSS Series B, 78, pp. 869–893.

**Value**

A list containing 'alp.hat' (estimate of the unknown component weight), 'Fs.hat' (list with elements 'x' and 'y' values for the function estimate of the unknown cumulative distribution function), 'dist.out' which is an object of the class 'dist.fun' using the complete data.gen, 'c.n' the value of the tuning parameter used to compute the final estimate, and finally 'cv.out' which is an object of class 'cv.mixmodel'. The object is NULL if method is "fixed".

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)



**Examples**

```
## Simulate data:
comp.dist <- list(f = 'norm', g = 'norm')
comp.param <- list(f = list(mean = 3, sd = 0.5),
                  g = list(mean = 0, sd = 1))
data1 <- rsimmix(n = 2000, unknownComp_weight = 0.3, comp.dist, comp.param)[['mixt.data']]
## Transform the known component of the admixture model into a Uniform(0,1) distribution:
comp.dist <- list(f = NULL, g = 'norm')
comp.param <- list(f = NULL, g = list(mean = 0, sd = 1))
data1_transfo <- knownComp_to_uniform(data = data1, comp.dist = list(comp.dist$f, comp.dist$g),
                                     comp.param = list(comp.param$f, comp.param$g))

plot(density(data1_transfo))
## Estimate the proportion of the unknown component of the admixture model:
PatraSen_cv_mixmodel(data = data1_transfo, folds = 3, reps = 1, cn.s = NULL,
                    cn.length = 3, gridsize = 100)$alp.hat
```

---

PatraSen\_density\_est    *Compute the estimate of the density of the unknown component in an admixture model*

---

**Description**

Compute by Patra and Sen technique the estimate of f.s (density corresponding to F.s) when f.s is known to be either decreasing or increasing.

**Usage**

```
PatraSen_density_est(input, dec.density = TRUE)
```

**Arguments**

input                    an R object of class 'cv.mixmodel' or 'mixmodel'.  
dec.density              a boolean indicating whether the density is increasing or decreasing.

**Details**

See Patra, R.K. and Sen, B. (2016); Estimation of a Two-component Mixture Model with Applications to Multiple Testing; JRSS Series B, 78, pp. 869–893.

**Value**

an estimator of the unknown component density.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```

comp.dist <- list(f = 'norm', g = 'norm')
comp.param <- list(f = list(mean = 3, sd = 0.5), g = list(mean = 0, sd = 1))
data1 <- rsimmix(n = 2000, unknownComp_weight = 0.6, comp.dist, comp.param)[['mixt.data']]
data1_transfo <- knownComp_to_uniform(data = data1, comp.dist = list(comp.dist$f, comp.dist$g),
                                     comp.param = list(comp.param$f, comp.param$g))
res <- PatraSen_cv_mixmodel(data = data1_transfo, folds = 3, reps = 1, cn.s = NULL,
                           cn.length = 3, gridsize = 200)
PatraSen_density_est(res, dec.density = TRUE)

```

---

PatraSen_dist_calc	<i>Compute the distance to be minimized using Patra and Sen estimation technique in admixture models</i>
--------------------	--

---

**Description**

Compute the distance to be minimized using Patra and Sen estimation technique by integrating along some given grid the appropriate distance. For further developments, see 'Details' below.

**Usage**

```
PatraSen_dist_calc(data, gridsize = 200)
```

**Arguments**

data	Sample where the known component density of the admixture model has been transformed into a Uniform(0,1) distribution.
gridsize	Gridsize to make the computations.

**Details**

See Patra, R.K. and Sen, B. (2016); Estimation of a Two-component Mixture Model with Applications to Multiple Testing; JRSS Series B, 78, pp. 869–893.

**Value**

a list containing the evaluated distance and some additional information.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```

comp.dist <- list(f = 'norm', g = 'norm')
comp.param <- list(f = list(mean = 3, sd = 0.5), g = list(mean = 0, sd = 1))
data1 <- rsimmix(n = 3000, unknownComp_weight = 0.6, comp.dist, comp.param)[['mixt.data']]
data1_transfo <- knownComp_to_uniform(data = data1, comp.dist = list(comp.dist$f, comp.dist$g),
                                     comp.param = list(comp.param$f, comp.param$g))
PatraSen_dist_calc(data = data1_transfo, gridsize = 200)

```

---

PatraSen\_est\_mix\_model

*Estimate by Patra and Sen the unknown component weight as well as the unknown distribution in admixture models*

---

**Description**

Estimation of unknown elements (by Patra and Sen method) under the admixture model with probability density function  $l = p*f + (1-p)*g$ , where  $g$  is the known component of the two-component mixture,  $p$  is the unknown proportion of the unknown component distribution  $f$ . More information in 'Details' below concerning the estimation method.

**Usage**

```

PatraSen_est_mix_model(
  data,
  method = c("lwr.bnd", "fixed", "cv"),
  c.n = NULL,
  folds = 10,
  reps = 1,
  cn.s = NULL,
  cn.length = 100,
  gridsize = 600
)

```

**Arguments**

<code>data</code>	Sample where the known component density of the admixture model has been transformed into a Uniform(0,1) distribution.
<code>method</code>	Either 'fixed' or 'cv', depending on whether compute the estimate based on the value of 'c.n' or use cross-validation for choosing 'c.n' (tuning parameter).
<code>c.n</code>	A positive number, with default value equal to $0.1 \log(\log(n))$ , where 'n' is the length of the observed sample.
<code>folds</code>	Number of folds used for cross-validation, default is 10.
<code>reps</code>	Number of replications for cross-validation, default is 1.

cn.s	A sequence of 'c.n' to be used for cross-validation (vector of values). Default is equally spaced grid of 100 values between $.001 \times \log(\log(n))$ and $0.2 \times \log(\log(n))$ .
cn.length	(default to 100) Number of equally spaced tuning parameter (between $.001 \times \log(\log(n))$ and $0.2 \times \log(\log(n))$ ). Values to search from.
gridsize	(default to 600) Number of equally spaced points (between 0 and 1) to evaluate the distance function. Larger values are more computationally intensive but also lead to more accurate estimates.

### Details

See Patra, R.K. and Sen, B. (2016); Estimation of a Two-component Mixture Model with Applications to Multiple Testing; JRSS Series B, 78, pp. 869–893.

### Value

A list containing 'alp.hat' (estimate of the unknown component weight), 'Fs.hat' (list with elements 'x' and 'y' values for the function estimate of the unknown cumulative distribution function), 'dist.out' which is an object of the class 'dist.fun' using the complete data.gen, 'c.n' the value of the tuning parameter used to compute the final estimate, and finally 'cv.out' which is an object of class 'cv.mixmodel'. The object is NULL if method is "fixed".

### Author(s)

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

### Examples

```
## Simulate data:
list.comp <- list(f = 'norm', g = 'norm')
list.param <- list(f = list(mean = 3, sd = 0.5),
                 g = list(mean = 0, sd = 1))
data1 <- rsimmix(n = 1500, unknownComp_weight = 0.8, list.comp, list.param)[['mixt.data']]
## Transform the known component of the admixture model into a Uniform(0,1) distribution:
list.comp <- list(f = NULL, g = 'norm')
list.param <- list(f = NULL, g = list(mean = 0, sd = 1))
data1_transfo <- knownComp_to_uniform(data = data1, comp.dist=list.comp, comp.param=list.param)
PatraSen_est_mix_model(data = data1_transfo, method = 'fixed',
                      c.n = 0.1*log(log(length(data1_transfo))), gridsize = 1000)$alp.hat
```

---

plot.decontaminated\_density

*Plot the decontaminated density of the unknown component for an estimated admixture model*

---

**Description**

Plot the decontaminated density of the unknown component in the admixture model under study, after inversion of the admixture cumulative distribution function. Recall that an admixture model follows the cumulative distribution function (CDF)  $L$ , where  $L = p \cdot F + (1-p) \cdot G$ , with  $g$  a known CDF and  $p$  and  $f$  unknown quantities.

**Usage**

```
## S3 method for class 'decontaminated_density'
plot(x, ..., x_val, add_plot = FALSE)
```

**Arguments**

<code>x</code>	An object of class 'decontamin_dens' (see ?decontaminated_density).
<code>...</code>	Arguments to be passed to methods, such as graphical parameters (see par).
<code>x_val</code>	A vector of X-axis values at which to plot the decontaminated density $f$ .
<code>add_plot</code>	(default to FALSE) A boolean specifying if one plots the decontaminated density over an existing plot. Used for visual comparison purpose.

**Details**

The decontaminated density is obtained by inverting the admixture density, given by  $l = p \cdot f + (1-p) \cdot g$ , to isolate the unknown component  $f$  after having estimated  $p$ .

**Value**

The plot of the decontaminated density.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
##### Continuous support:
## Simulate data:
list.comp <- list(f1 = 'norm', g1 = 'norm',
                 f2 = 'norm', g2 = 'norm')
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 3, sd = 0.5), g2 = list(mean = 5, sd = 2))
sample1 <- rsimmix(n=3000, unknownComp_weight=0.7, comp.dist = list(list.comp$f1, list.comp$g1),
                  comp.param=list(list.param$f1, list.param$g1))
sample2 <- rsimmix(n=2500, unknownComp_weight=0.8, comp.dist = list(list.comp$f2, list.comp$g2),
                  comp.param=list(list.param$f2, list.param$g2))

## Estimate the mixture weight in each of the sample in real-life setting:
list.comp <- list(f1 = NULL, g1 = 'norm',
                 f2 = NULL, g2 = 'norm')
list.param <- list(f1 = NULL, g1 = list(mean = 0, sd = 1),
                  f2 = NULL, g2 = list(mean = 5, sd = 2))
estimate <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], comp.dist = list.comp,
```

```

        comp.param = list.param, with.correction = FALSE, n.integ = 1000)
## Determine the decontaminated version of the unknown density by inversion:
res1 <- decontaminated_density(sample1 = sample1[['mixt.data']], comp.dist = list.comp[1:2],
    comp.param = list.param[1:2], estim.p = estimate$prop.estim[1])
res2 <- decontaminated_density(sample1 = sample2[['mixt.data']], comp.dist = list.comp[3:4],
    comp.param = list.param[3:4], estim.p = estimate$prop.estim[2])
## Use appropriate sequence of x values:
plot(x = res1, x_val = seq(from = 0, to = 6, length.out = 100), add_plot = FALSE)
plot(x = res2, col = "red", x_val = seq(from = 0, to = 6, length.out = 100), add_plot = TRUE)

##### Countable discrete support:
list.comp <- list(f1 = 'pois', g1 = 'pois',
    f2 = 'pois', g2 = 'pois')
list.param <- list(f1 = list(lambda = 3), g1 = list(lambda = 2),
    f2 = list(lambda = 3), g2 = list(lambda = 4))
sample1 <- rsimmix(n=4000, unknownComp_weight=0.7, comp.dist = list(list.comp$f1,list.comp$g1),
    comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=3500, unknownComp_weight=0.85, comp.dist = list(list.comp$f2,list.comp$g2),
    comp.param=list(list.param$f2,list.param$g2))
## Estimate the mixture weight in each of the sample in real-life setting:
list.comp <- list(f1 = NULL, g1 = 'pois',
    f2 = NULL, g2 = 'pois')
list.param <- list(f1 = NULL, g1 = list(lambda = 2),
    f2 = NULL, g2 = list(lambda = 4))
estimate <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], comp.dist = list.comp,
    comp.param = list.param, with.correction = FALSE, n.integ = 1000)
## Determine the decontaminated version of the unknown density by inversion:
res1 <- decontaminated_density(sample1 = sample1[['mixt.data']], comp.dist = list.comp[1:2],
    comp.param = list.param[1:2], estim.p = estimate$prop.estim[1])
res2 <- decontaminated_density(sample1 = sample2[['mixt.data']], comp.dist = list.comp[3:4],
    comp.param = list.param[3:4], estim.p = estimate$prop.estim[2])
## Use appropriate sequence of x values:
plot(x = res1, x_val = seq(from = 0, to = 15, by = 1), add_plot = FALSE)
plot(x = res2, col = "red", x_val= seq(from=0,to=15,by=1), add_plot = TRUE)

##### Finite discrete support:
list.comp <- list(f1 = 'multinom', g1 = 'multinom',
    f2 = 'multinom', g2 = 'multinom')
list.param <- list(f1 = list(size=1, prob=c(0.3,0.4,0.3)), g1 = list(size=1, prob=c(0.6,0.3,0.1)),
    f2 = list(size=1, prob=c(0.3,0.4,0.3)), g2 = list(size=1, prob=c(0.2,0.6,0.2)))
sample1 <- rsimmix(n=4000, unknownComp_weight=0.8, comp.dist = list(list.comp$f1,list.comp$g1),
    comp.param=list(list.param$f1,list.param$g1))
sample2 <- rsimmix(n=3500, unknownComp_weight=0.9, comp.dist = list(list.comp$f2,list.comp$g2),
    comp.param=list(list.param$f2,list.param$g2))
## Estimate the mixture weight in each of the sample in real-life setting:
list.comp <- list(f1 = NULL, g1 = 'multinom',
    f2 = NULL, g2 = 'multinom')
list.param <- list(f1 = NULL, g1 = list(size=1, prob=c(0.6,0.3,0.1)),
    f2 = NULL, g2 = list(size=1, prob=c(0.2,0.6,0.2)))
estimate <- IBM_estimProp(sample1[['mixt.data']], sample2[['mixt.data']], comp.dist = list.comp,
    comp.param = list.param, with.correction = FALSE, n.integ = 1000)
## Determine the decontaminated version of the unknown density by inversion:
res1 <- decontaminated_density(sample1 = sample1[['mixt.data']], comp.dist = list.comp[1:2],

```

```

                                comp.param = list.param[1:2], estim.p = estimate$prop.estim[1])
res2 <- decontaminated_density(sample1 = sample2[['mixt.data']], comp.dist = list.comp[3:4],
                                comp.param = list.param[3:4], estim.p = estimate$prop.estim[2])
## Use appropriate sequence of x values:
plot(x = res1, x_val = seq(from = 0, to=6, by = 1), add_plot = FALSE)
plot(x = res2, col = "red", x_val = seq(from = 0, to = 6, by = 1), add_plot = TRUE)

```

---

plot\_mixt\_density      *Plot the density of some given sample(s) with mixture distributions.*

---

## Description

Plot the density of the sample(s) with optional arguments to improve the visualization.

## Usage

```

plot_mixt_density(
  samples,
  user.bounds = NULL,
  support = c("continuous", "discrete"),
  main = ""
)

```

## Arguments

samples	Observed samples (mixture distributions) from which the density will be plotted.
user.bounds	(default to NULL) Bounds to limit the range of x-axis when plotting.
support	Support of the distributions, to know whether density plot or histogram should be displayed.
main	Title for the plot.

## Value

a plot with the densities of the samples provided as inputs.

## Author(s)

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

## Examples

```

##### Continuous support:
list.comp <- list(f1 = "norm", g1 = "norm",
                 f2 = "norm", g2 = "norm",
                 f3 = "norm", g3 = "norm")
list.param <- list(f1 = list(mean = 5, sd = 1), g1 = list(mean = 2, sd = 0.7),
                  f2 = list(mean = 0, sd = 1), g2 = list(mean = -3, sd = 1.1),

```

```

      f3 = list(mean = 9, sd = 1), g3 = list(mean = 6, sd = 2))
## Simulate data:
sim1 <- rsimmix(n = 300, unknownComp_weight = 0.8, comp.dist = list(list.comp$f1,list.comp$g1),
  comp.param = list(list.param$f1, list.param$g1))$mixt.data
sim2 <- rsimmix(n= 250, unknownComp_weight = 0.85, comp.dist = list(list.comp$f2,list.comp$g2),
  comp.param = list(list.param$f2, list.param$g2))$mixt.data
sim3 <- rsimmix(n= 400, unknownComp_weight = 0.6, comp.dist = list(list.comp$f3,list.comp$g3),
  comp.param = list(list.param$f3, list.param$g3))$mixt.data
plot_mixt_density(samples = list(sim1,sim2,sim3), user.bounds = NULL, support = "continuous")

##### Countable discrete support:
list.comp <- list(f1 = 'pois', g1 = 'pois',
  f2 = 'pois', g2 = 'pois')
list.param <- list(f1 = list(lambda = 7), g1 = list(lambda = 1),
  f2 = list(lambda = 2), g2 = list(lambda = 15))
sim1 <- rsimmix(n=4000, unknownComp_weight=0.5, comp.dist = list(list.comp$f1,list.comp$g1),
  comp.param=list(list.param$f1,list.param$g1))$mixt.data
sim2 <- rsimmix(n=3500, unknownComp_weight=0.3, comp.dist = list(list.comp$f2,list.comp$g2),
  comp.param=list(list.param$f2,list.param$g2))$mixt.data
plot_mixt_density(samples = list(sim1,sim2), user.bounds = NULL, support = "discrete")

```

---

poly\_orthonormal\_basis

*Build an orthonormal basis to decompose some given probability density function*

---

## Description

Build an orthonormal basis, needed to decompose the probability density function (pdf) of the unknown component from the admixture, depending on the support under consideration.

## Usage

```

poly_orthonormal_basis(
  support = c("Real", "Integer", "Positive", "Bounded.continuous", "Bounded.discrete"),
  deg,
  x,
  m
)

```

## Arguments

support	Support of the random variables implied in the two-component mixture distribution.
deg	Degree up to which the basis is built.
x	(NULL by default) Only used when support is 'Integer'. The point at which the polynomial value will be evaluated.
m	(NULL by default) Only used when support is 'Integer'. Corresponds to the mean of the reference measure, i.e. Poisson(m).



**Value**

the orthonormal polynomial basis used to decompose the density of the unknown component of the mixture distribution.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
poly_orthonormal_basis(support = 'Real', deg = 10, x = NULL, m = NULL)
```

---

```
print.admix_cluster
```

*Results of the clustering algorithm performed over the K populations following admixture models.*

---

**Description**

Print the detected clusters among the populations under study. This method also prints the number of clusters, the p-values of statistical tests performed when building the clusters, the estimated weights of the unknown component distributions inside each cluster, and the discrepancy matrix. The latter represents some kind of distance between the populations.

**Usage**

```
## S3 method for class 'admixture_cluster'
print(x, ...)
```

**Arguments**

x                    An object of class 'admixture\_cluster' (see ?admixture\_clustering).  
 ...                    further arguments passed to or from other methods.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
## Simulate data (chosen parameters indicate 2 clusters (populations (1,3), (2,4))!):
list.comp <- list(f1 = "gamma", g1 = "exp",
                 f2 = "gamma", g2 = "exp",
                 f3 = "gamma", g3 = "gamma",
                 f4 = "gamma", g4 = "exp")
list.param <- list(f1 = list(shape = 16, rate = 4), g1 = list(rate = 1/3.5),
                  f2 = list(shape = 14, rate = 2), g2 = list(rate = 1/5),
```

```

      f3 = list(shape = 16, rate = 4), g3 = list(shape = 12, rate = 2),
      f4 = list(shape = 14, rate = 2), g4 = list(rate = 1/7))
A.sim <- rsimmix(n=2600, unknownComp_weight=0.8, comp.dist = list(list.comp$f1,list.comp$g1),
  comp.param = list(list.param$f1, list.param$g1))$mixt.data
B.sim <- rsimmix(n=3000, unknownComp_weight=0.7, comp.dist = list(list.comp$f2,list.comp$g2),
  comp.param = list(list.param$f2, list.param$g2))$mixt.data
C.sim <- rsimmix(n=3500, unknownComp_weight=0.6, comp.dist = list(list.comp$f3,list.comp$g3),
  comp.param = list(list.param$f3, list.param$g3))$mixt.data
D.sim <- rsimmix(n=4800, unknownComp_weight=0.5, comp.dist = list(list.comp$f4,list.comp$g4),
  comp.param = list(list.param$f4, list.param$g4))$mixt.data
## Look for the clusters:
list.comp <- list(f1 = NULL, g1 = "exp",
  f2 = NULL, g2 = "exp",
  f3 = NULL, g3 = "gamma",
  f4 = NULL, g4 = "exp")
list.param <- list(f1 = NULL, g1 = list(rate = 1/3.5),
  f2 = NULL, g2 = list(rate = 1/5),
  f3 = NULL, g3 = list(shape = 12, rate = 2),
  f4 = NULL, g4 = list(rate = 1/7))
clusters <- admix_clustering(samples = list(A.sim,B.sim,C.sim,D.sim), n_sim_tab = 8,
  comp.dist=list.comp, comp.param=list.param, conf.level = 0.95,
  parallel=FALSE, n_cpu=2)
print(clusters)

```

---

```
print.admix_estim      Print the results of estimated parameters from K admixture models
```

---

## Description

Print the estimated weight  $p$  of the unknown component in the admixture model under study Recall that an admixture model follows the cumulative distribution function (CDF)  $L$ , where  $L = p * F + (1-p) * G$ , with  $g$  a known CDF and  $p$  and  $f$  unknown quantities.

## Usage

```
## S3 method for class 'admixture_estim'
print(x, ...)
```

## Arguments

```
x          An object of class 'admixture_estim' (see ?admixture_estim).
...        further arguments passed to or from other methods.
```

## Author(s)

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
##### On a simulated example to see whether the true parameters are well estimated.
list.comp <- list(f1 = "norm", g1 = "norm",
                 f2 = "norm", g2 = "norm")
list.param <- list(f1 = list(mean = 0, sd = 1), g1 = list(mean = 2, sd = 0.7),
                  f2 = list(mean = 0, sd = 1), g2 = list(mean = -3, sd = 1.1))
## Simulate data:
sim1 <- rsimmix(n = 2100, unknownComp_weight = 0.8, comp.dist = list(list.comp$f1, list.comp$g1),
               comp.param = list(list.param$f1, list.param$g1))$mixt.data
sim2 <- rsimmix(n = 2000, unknownComp_weight = 0.85, comp.dist = list(list.comp$f2, list.comp$g2),
               comp.param = list(list.param$f2, list.param$g2))$mixt.data
## Estimate the mixture weights of the admixture models:
list.comp <- list(f1 = NULL, g1 = "norm",
                 f2 = NULL, g2 = "norm")
list.param <- list(f1 = NULL, g1 = list(mean = 2, sd = 0.7),
                  f2 = NULL, g2 = list(mean = -3, sd = 1.1))
estim <- admix_estim(samples = list(sim1, sim2), sym.f = TRUE, est.method = 'IBM',
                    comp.dist = list.comp, comp.param = list.param)
print(x = estim)
```

---

print.admix_test	<i>Print the results of statistical test for equality of unknown component distributions in admixture models</i>
------------------	--

---

**Description**

Print the decision (as well as other useful information) of the statistical test with null hypothesis corresponding to the equality of unknown component distributions in admixture models. More precisely, given two (or more) admixture models with cumulative distribution functions (CDF)  $L_1$  and  $L_2$ , where  $L_i = \pi_i F_i + (1 - \pi_i) G_i$   $i=1,2$  and  $G_i$  are the known CDFs, the function performs the test:  $H_0: F_1 = F_2$  versus  $H_1: F_1 \neq F_2$ .

**Usage**

```
## S3 method for class 'admixture_test'
print(x, ...)
```

**Arguments**

**x** An object of class 'admixture\_test' (see ?admixture\_test).  
**...** further arguments passed to or from other methods.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

## Examples

```
##### On a simulated example, with 1 sample (gaussianity test):
list.comp <- list(f1 = "norm", g1 = "norm")
list.param <- list(f1 = list(mean = 0, sd = 1), g1 = list(mean = 2, sd = 0.7))
## Simulate data:
sim1 <- rsimmix(n = 300, unknownComp_weight = 0.85, comp.dist = list(list.comp$f1, list.comp$g1),
               comp.param = list(list.param$f1, list.param$g1))$mixt.data
## Perform the test hypothesis:
list.comp <- list(f1 = NULL, g1 = "norm")
list.param <- list(f1 = NULL, g1 = list(mean = 2, sd = 0.7))
gaussTest <- admix_test(samples = list(sim1), sym.f = TRUE, test.method = 'Poly', sim_U = NULL,
                       n_sim_tab = 50, comp.dist = list.comp, comp.param = list.param,
                       support = "Real", conf.level = 0.95, parallel = FALSE, n_cpu = 2)

print(gaussTest)
```

---

 rsimmix

*Simulation of a two-component mixture model*


---

## Description

Simulate a two-component mixture model following the probability density function (pdf)  $l$  such that  $l = p*f + (1-p)*g$ , with  $f$  and  $g$  mixture component distributions, and  $p$  the mixture weight.

## Usage

```
rsimmix(
  n = 1000,
  unknownComp_weight = 0.5,
  comp.dist = list(f = "norm", g = "norm"),
  comp.param = list(f = c(mean = 0, sd = 1), g = c(mean = 2, sd = 1))
)
```

## Arguments

<code>n</code>	Number of observations to be drawn.
<code>unknownComp_weight</code>	Weight of the component distribution $f$ (representing the unknown component in admixture models).
<code>comp.dist</code>	A list with two elements corresponding to the component distributions (specified with R native names for these distributions) involved in the mixture model. These elements respectively refer to the two components $f$ and $g$ . No unknown elements permitted. For instance, 'comp.dist' could be set equal to <code>list(f = 'norm', g = 'norm')</code> .
<code>comp.param</code>	A list with two elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must

correspond to the native R argument names for these distributions. These elements respectively refer to the parameters of f and g distributions of the mixture model. No unknown elements permitted. For instance, 'comp.param' could be set equal to `list(f=list(mean=2,sd=0.3), g=list(mean=0,sd=1))`.

### Value

A list of three components. The first, named 'mixt.data', is the simulated sample from the specified mixture distribution. The second, named 'unknown.data', refers to the data simulated corresponding to the distribution f. The third, named 'known.data', corresponds to the observations affiliated to the known component g.

### Author(s)

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

### Examples

```
sim.X <- rsimmix(n = 2000, unknownComp_weight = 0.7, comp.dist = list(f = 'norm', g = 'norm'),
                comp.param = list(f = list(mean = 3, sd = 0.5), g = list(mean = 0, sd = 1)))
class(sim.X)
attributes(sim.X)
plot_mixt_density(samples = list(sim.X$mixt.data), user.bounds = NULL, support = 'continuous')
```

---

rsimmix\_mix

*Simulation of a two-component gaussian mixture with one component following a two-component gaussian mixture*

---

### Description

Simulate a two-component gaussian admixture model, where the first component is a gaussian mixture itself

### Usage

```
rsimmix_mix(n, m, s, p, a)
```

### Arguments

n	is the number of observations to be drawn
m	the mean (up to the shift a) of the unknown components
s	the standard deviation of the unknown components
p	the weight of the unknown component (itself a mixture).
a	the shift of the mean for the two distributions that are embedded in the unknown component

**Value**

a list containing the data generated from a mixture of mixture distribution, the data where the known component density has been made uniform(0,1), and the known data (corresponding to the part of data generated from the known component density).

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
sample1 <- rsimmix_mix(n = 3000, m = 5, s = 0.5, p = 0.3, a = 2)[['mixt.data']]
plot(stats::density(sample1))
```

---

sim\_gaussianProcess     *Simulation of a Gaussian process*

---

**Description**

Simulate the trajectory of a Gaussian process, given a mean vector and a variance-covariance structure.

**Usage**

```
sim_gaussianProcess(
  mean_vec,
  varCov_mat,
  from = 0,
  to = 1,
  start = 0,
  nb.points = 10
)
```

**Arguments**

mean_vec	Vector (if multidimensional) of means for the increments following gaussian distribution.
varCov_mat	Corresponding variance-covariance structure.
from	Initial time point at which the process is simulated.
to	Last time point at which the process is simulated.
start	Useful if the user wants to make the trajectory start from some given value.
nb.points	Number of points at which the process is simulated.

**Value**

The trajectory of the Gaussian processes after simulating the multivariate Gaussian distributions with specified variance-covariance structure.

**Author(s)**

Xavier Milhaud [xavier.milhaud.research@gmail.com](mailto:xavier.milhaud.research@gmail.com)

**Examples**

```
list.comp <- list(f1 = "norm", g1 = "norm")
list.param <- list(f1 = list(mean = 12, sd = 0.4),
                  g1 = list(mean = 16, sd = 0.7))
sample1 <- rsimmix(n = 2000, unknownComp_weight = 0.5, comp.dist = list.comp,
                  comp.param = list.param)$mixt.data
## First get the variance-covariance matrix of the empirical process (Donsker correlation):
cov_mat <- .Call('_admix_estimVarCov_empProcess_Rcpp', PACKAGE = 'admix',
                seq(from = min(sample1), to = max(sample1), length.out = 100), sample1)
## Plug it into the simulation of the gaussian process:
B1 <- sim_gaussianProcess(mean_vec=rep(0,nrow(cov_mat)), varCov_mat=cov_mat, from=min(sample1),
                          to = max(sample1), start = 0, nb.points = nrow(cov_mat))
plot(x = B1$dates, y = B1$traj1, type="l", xlim = c(min(sample1),max(sample1)), ylim = c(-1,1))
```

---

stmf\_small

*Short-term Mortality Fluctuations (STMF) data series, restricted to 6 countries (Belgium, France, Italy, Netherlands, Spain, Germany).*

---

**Description**

Weekly death counts provide the most objective and comparable way of assessing the scale of short-term mortality elevations across countries (32 countries) and time. Extraction date: 09/21/2020.

**Usage**

```
stmf_small
```

**Format**

A data frame with 88146 rows and 19 variables:

**CountryCode** Mortality database country code

**Year** Year

**Week** Week number

**Sex** Gender ('m': male, 'f': female, 'b': both)

**D0\_14** Age range 0-14

**D15\_64** Age range 15-64

- D65\_74** Age range 65-74
- D75\_84** Age range 75-84
- D85p** Age range 85+
- DTotal** Count of deaths for all ages combined
- R0\_14** Crude death rate for age range 0-14
- R15\_64** Crude death rate for age range 15-64
- R65\_74** Crude death rate for age range 65-74
- R75\_84** Crude death rate for age range 75-84
- R85p** Crude death rate for age range 85+
- RTotal** Crude death rate for all ages combined
- Split** Indicates if data were split from aggregated age groups (0 if the original data has necessary detailed age scale). For example, if the original age scale was 0-4, 5-29, 30-65, 65+, then split will be equal to 1
- SplitSex** Indicates if the original data are available by sex (0) or data are interpolated (1)
- Forecast** Equals 1 for all years where forecasted population exposures were used to calculate weekly death rates

### Source

<https://www.mortality.org>

---

two_samples_test	<i>Two-samples hypothesis test on the unknown component in admixture models</i>
------------------	---

---

### Description

Test hypothesis on the unknown component of admixture models using different estimation techniques, and different testing strategies.

### Usage

```
two_samples_test(
  samples,
  known.p = NULL,
  comp.dist = NULL,
  comp.param = NULL,
  method = c("ICV", "Poly"),
  n_sim_tab = NULL,
  K = 3,
  support = c("Real", "Positive", "Integer", "Bounded.continuous"),
  est.method = c("BVdk", "PS"),
  s = 0.49,
  nb.ssEch = 2,
```



```

var.explicit = F,
nb.echBoot = NULL,
bounds.supp = NULL,
parallel = FALSE,
n_cpu = 2
)

```

### Arguments

samples	A list of the two observed samples, where each sample follows the mixture distribution given by $l = p*f + (1-p)*g$ , with $f$ and $p$ unknown and $g$ known.
known.p	(default to NULL) The true component weights $p_1$ and $p_2$ if known, only useful in simulation studies.
comp.dist	A list with four elements corresponding to the component distributions (specified with R native names for these distributions) involved in the two admixture models. The two first elements refer to the unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.dist' could be specified as follows: <code>list(f1=NULL, g1='norm', f2=NULL, g2='norm')</code> .
comp.param	A list with four elements corresponding to the parameters of the component distributions, each element being a list itself. The names used in this list must correspond to the native R argument names for these distributions. The two first elements refer to the parameters of unknown and known components of the 1st admixture model, and the last two ones to those of the second admixture model. If there are unknown elements, they must be specified as 'NULL' objects. For instance, 'comp.param' could be specified as follows: <code>: list(f1=NULL, g1=list(mean=0,sd=1), f2=NULL, g2=list(mean=3,sd=1.1))</code> .
method	Method used for testing. Choose either 'Poly' or 'ICV'. 'Poly' refers to comparison of expansion coefficients in polynomial orthonormal basis, whereas 'ICV' refers to the Inner Convergence property obtained when using the IBM approach. More details are provided below in 'Details'.
n_sim_tab	(Only with 'ICV' method) Number of simulated gaussian processes used for the tabulation of the Inner Convergence distribution in IBM approach.
K	(Only for 'Poly' method) Number of coefficients considered for the polynomial basis expansion.
support	(Only for 'Poly' method) Support of the densities under consideration, useful to choose the polynomial orthonormal basis. One of 'Real', 'Integer', 'Positive', or 'Bounded.continuous'.
est.method	(Only for 'Poly' method) Either 'BVdk' (Bordes and Valdekerkhove estimation technique) or 'PS' (Patra and Sen estimation technique). The latter should not be used since the estimators plugged into the test statistic are not square-root $n$ consistent. More details are given in Section 'Details' below.
s	(Only for 'Poly' method) Rate at which the normalization factor is set in the penalization rule for model selection (in $]0,1/2[$ ).

<code>nb.ssEch</code>	(Only with 'Poly' method) Number of subsamples created from original data to decorrelate the estimation of the parameters.
<code>var.explicit</code>	(Only with 'Poly' method) Boolean that enables to choose between explicit evaluation of the variance of the test statistic or not (FALSE=bootstrap). FIXME: it seems that bootstrap procedure does not work in the context of admixtures.
<code>nb.echBoot</code>	(Only with 'Poly' method) Number of bootstrap samples if 'var.explicit' is set to FALSE.
<code>bounds.supp</code>	(Only with 'Poly' method) default to NULL. Useful if support = 'bounded.continuous', a list of minimum and maximum bounds, specified as follows: <code>list( list(min.f1,min.g1,min.f2,min.g2) , list(max.f1,max.g1,max.f2,max.g2) )</code>
<code>parallel</code>	Boolean to indicate whether parallel computations are performed (speed-up the tabulation).
<code>n_cpu</code>	Number of cores used when parallelizing.

### Details

Here as some details concerning the different methods that can be chosen: i) 'Poly' relies on two-sample testing strategy where each unknown component density is decomposed in an orthonormal polynomial basis, and the estimation of the component weights related to the two two-component admixture models can be performed either using Patra and Sen estimator (despite the latter is not square-root  $n$  consistent and thus should not be used in such hypothesis tests), or by Bordes and Vandekerkhove estimation technique (if the unknown component density is symmetric); ii) 'ICV' refers to Inversion - Best Matching strategy which has no constraints except that we need to handle two samples.

### Value

The decision of the test with further information such as p-value and others, depending on the method used.

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### Examples

```
##### Under the null hypothesis H0 :
## Simulate data:
list.comp <- list(f1 = "norm", g1 = "norm",
                 f2 = "norm", g2 = "norm")
list.param <- list(f1 = list(mean = 3, sd = 0.5), g1 = list(mean = 0, sd = 1),
                  f2 = list(mean = 3, sd = 0.5), g2 = list(mean = 6, sd = 1.2))
sample1 <- rsimmix(n=250, unknownComp_weight=0.85, comp.dist = list(list.comp$f1,list.comp$g1),
                  comp.param = list(list.param$f1,list.param$g1))[['mixt.data']]
sample2 <- rsimmix(n=300, unknownComp_weight=0.8, comp.dist = list(list.comp$f2,list.comp$g2),
                  comp.param = list(list.param$f2,list.param$g2))[['mixt.data']]
plot_mixt_density(samples = list(sample1,sample2), user.bounds=NULL, support='continuous')
##### Performs the test by the different methods :
```

```
list.comp <- list(f1 = NULL, g1 = "norm",
                 f2 = NULL, g2 = "norm")
list.param <- list(f1 = NULL, g1 = list(mean = 0, sd = 1),
                  f2 = NULL, g2 = list(mean = 6, sd = 1.2))
## Using expansion coefficients in orthonormal polynomial basis:
two_samples_test(samples = list(sample1, sample2), comp.dist=list.comp, comp.param=list.param,
                 method = 'Poly', K = 3, support = 'Real', est.method = 'BVdk', s = 0.4,
                 nb.ssEch = 2, var.explicit = TRUE)
```

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