

# Package ‘fntl’

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**Title** Numerical Tools for 'Rcpp' and Lambda Functions

**Version** 0.1.2

**Description** Provides a 'C++' API for routinely used numerical tools such as integration, root-finding, and optimization, where function arguments are given as lambdas. This facilitates 'Rcpp' programming, enabling the development of 'R'-like code in 'C++' where functions can be defined on the fly and use variables in the surrounding environment.

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fntl-package	<i>fntl</i>
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### Description

Package documentation

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### See Also

Useful links:

- <https://github.com/andrewraim/fntl>

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args	<i>Arguments</i>
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### Description

Get an arguments list for internal methods with the default settings. This object can be adjusted and passed to the respective function.

### Usage

findroot\_args()

optimize\_args()

integrate\_args()

cg\_args()

bfgs\_args()

lbfgsb\_args()

neldermead\_args()

nlm\_args()

richardson\_args()

### Value

An argument list corresponding to the specified function. The elements of the list are named and supplied with default values. See the package vignette for further details.

- `findroot_args` is documented in the section "Root-Finding".
- `optimize_args` is documented in the section "Univariate Optimization".
- `integrate_args` is documented in the section "Integration".
- `cg_args` is documented in the section "Conjugate Gradient".
- `bfgs_args` is documented in the section "BFGS".
- `lbfgsb_args` is documented in the section "L-BFGS-B".
- `neldermead_args` is documented in the section "Nelder-Mead".
- `nlm_args` is documented in the section "Newton-Type Algorithm for Nonlinear Optimization".
- `richardson_args` is documented in the section "Richardson Extrapolated Finite Differences".

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deriv

*Numerical Derivatives via Finite Differences*

---

### Description

Numerical Derivatives via Finite Differences

### Usage

`fd_deriv1(f, x, i, h, fd_type)`

`fd_deriv2(f, x, i, j, h_i, h_j, fd_type)`

`deriv1(f, x, i, args, fd_type)`

`deriv2(f, x, i, j, args, fd_type)`

**Arguments**

f	Function to differentiate.
x	Scalar at which to evaluate the derivative.
i	First coordinate to differentiate.
h	Step size in the first coordinate.
fd_type	Type of derivative: 0 for symmetric difference, 1 for forward difference, and 2 for backward difference.
j	Second coordinate to differentiate.
h_i	Step size in the first coordinate.
h_j	Step size in the second coordinate.
args	List of additional arguments from the function <code>richardson_args</code> .

**Value**

`fd_deriv1` and `fd_deriv2` return a single numeric value corresponding to the first and second derivative via finite differences. `deriv1` and `deriv2` return a list with the form of a `richardson_result` described in section "Richardson Extrapolated Finite Differences" of the package vignette.

**Examples**

```
args = richardson_args()

f = sin # Try 2nd derivatives of a univariate function
x0 = 0.5
print(-sin(x0)) ## Exact answer for f'(x0)

fd_deriv2(f, x0, i = 0, j = 0, h_i = 0.001, h_j = 0.001, fd_type = 0)
fd_deriv2(f, x0, i = 0, j = 0, h_i = 0.001, h_j = 0.001, fd_type = 1)
fd_deriv2(f, x0, i = 0, j = 0, h_i = 0.001, h_j = 0.001, fd_type = 2)

deriv2(f, x0, i = 0, j = 0, args, fd_type = 0)

# Try 2nd derivatives of a bivariate function
f = function(x) { sin(x[1]) + cos(x[2]) }
x0 = c(0.5, 0.25)

print(-sin(x0[1])) ## Exact answer for f_xx(x0)
print(-cos(x0[2])) ## Exact answer for f_yy(x0)
print(0) ## Exact answer for f_xy(x0,y0)

numDeriv::hessian(f, x0)

fd_deriv2(f, x0, i = 0, j = 0, h_i = 0.001, h_j = 0.001, fd_type = 0)
fd_deriv2(f, x0, i = 0, j = 0, h_i = 0.001, h_j = 0.001, fd_type = 1)
fd_deriv2(f, x0, i = 0, j = 0, h_i = 0.001, h_j = 0.001, fd_type = 2)

fd_deriv2(f, x0, i = 0, j = 1, h_i = 0.001, h_j = 0.001, fd_type = 0)
fd_deriv2(f, x0, i = 0, j = 1, h_i = 0.001, h_j = 0.001, fd_type = 1)
```

```

fd_deriv2(f, x0, i = 0, j = 1, h_i = 0.001, h_j = 0.001, fd_type = 2)

fd_deriv2(f, x0, i = 1, j = 1, h_i = 0.001, h_j = 0.001, fd_type = 0)
fd_deriv2(f, x0, i = 1, j = 1, h_i = 0.001, h_j = 0.001, fd_type = 1)
fd_deriv2(f, x0, i = 1, j = 1, h_i = 0.001, h_j = 0.001, fd_type = 2)

deriv2(f, x0, i = 1, j = 1, args, fd_type = 0)
deriv2(f, x0, i = 1, j = 1, args, fd_type = 1)
deriv2(f, x0, i = 1, j = 1, args, fd_type = 2)

```

---

findroot

*Find Root*


---

## Description

Find Root

## Usage

```
findroot_bisect(f, lower, upper, args)
```

```
findroot_brent(f, lower, upper, args)
```

## Arguments

f	Function for which a root is desired.
lower	Lower limit of search interval. Must be finite.
upper	Upper limit of search interval. Must be finite.
args	List of additional arguments from the function <code>findroot_args</code> .

## Value

A list with the form of a `findroot_result` described in section "Root-Finding" of the package vignette.

## Examples

```

f = function(x) { x^2 - 1 }
args = findroot_args()
findroot_bisect(f, 0, 10, args)
findroot_brent(f, 0, 10, args)

```

---

gradient0                      *Numerical Gradient Vector*

---

**Description**

Numerical Gradient Vector

**Usage**

```
gradient0(f, x, args)
```

**Arguments**

f	Function to differentiate.
x	Vector at which to evaluate the gradient.
args	List of additional arguments from the function richardson_args.

**Value**

A list with the form of a `gradient_result` described in section "Gradient" of the package vignette.

**Examples**

```
f = function(x) { sum(sin(x)) }  
args = richardson_args()  
x0 = seq(0, 1, length.out = 5)  
cos(x0) ## Exact answer  
gradient0(f, x0, args)  
numDeriv::grad(f, x0)
```

---

hessian0                      *Numerical Hessian*

---

**Description**

Numerical Hessian

**Usage**

```
hessian0(f, x, args)
```

**Arguments**

f	Function to differentiate.
x	Vector at which to evaluate the Hessian.
args	List of additional arguments from the function richardson_args.

**Value**

A list with the form of a `hessian_result` described in section "Hessian" of the package vignette.

**Examples**

```
f = function(x) { sum(x^2) }
x0 = seq(1, 10, length.out = 5)
args = richardson_args()
hessian0(f, x0, args)
numDeriv::hessian(f, x0)
```

---

integrate0

*Integration*


---

**Description**

Compute the integral  $\int_a^b f(x)dx$ .

**Usage**

```
integrate0(f, lower, upper, args)
```

**Arguments**

<code>f</code>	Function to integrate.
<code>lower</code>	Lower limit of integral.
<code>upper</code>	Upper limit of integral.
<code>args</code>	List of additional arguments from the function <code>integrate_args</code> .

**Value**

A list with the form of a `integrate_result` described in section "Integration" of the package vignette.

**Examples**

```
f = function(x) { exp(-x^2 / 2) }
args = integrate_args()
integrate0(f, 0, 10, args)
```

jacobian0

*Numerical Jacobian Matrix*

---

**Description**

Numerical Jacobian Matrix

**Usage**`jacobian0(f, x, args)`**Arguments**

<code>f</code>	Function to differentiate.
<code>x</code>	Vector at which to evaluate the Jacobian.
<code>args</code>	List of additional arguments from the function <code>richardson_args</code> .

**Value**A list with the form of a `jacobian_result` described in section "Jacobian" of the package vignette.**Examples**

```
f = function(x) { cumsum(sin(x)) }
x0 = seq(1, 10, length.out = 5)
args = richardson_args()
out = jacobian0(f, x0, args)
print(out$value)
numDeriv::jacobian(f, x0)
```

---

matrix\_apply*Matrix Apply Functions*

---

**Description**

Matrix Apply Functions

**Usage**`mat_apply(X, f)``row_apply(X, f)``col_apply(X, f)`



**Arguments**

X	A matrix
f	The function to apply.

**Details**

The `mat_apply`, `row_apply`, and `col_apply` C++ functions are intended to operate like the following calls in R, respectively.

```
apply(x, c(1,2), f)
apply(x, 1, f)
apply(x, 2, f)
```

The R functions exposed here are specific to numeric-valued matrices, but the underlying C++ functions are intended to work with any type of Rcpp Matrix.

**Value**

`mat_apply` returns a matrix. `row_apply` and `col_apply` return a vector. See section "Apply" of the package vignette for details.

**Examples**

```
X = matrix(1:12, nrow = 4, ncol = 3)
mat_apply(X, f = function(x) { x^(1/3) })
row_apply(X, f = function(x) { sum(x^2) })
col_apply(X, f = function(x) { sum(x^2) })
```

---

multivariate-optimization

*Multivariate Optimization*

---

**Description**

Multivariate Optimization

**Usage**

```
cg1(init, f, g, args)
cg2(init, f, args)
bfgs1(init, f, g, args)
bfgs2(init, f, args)
```

```
lbfgsb1(init, f, g, args)
```

```
lbfgsb2(init, f, args)
```

```
neldermead(init, f, args)
```

```
nlm1(init, f, g, h, args)
```

```
nlm2(init, f, g, args)
```

```
nlm3(init, f, args)
```

### Arguments

<code>init</code>	Initial value
<code>f</code>	Function $f$ to optimize
<code>g</code>	Gradient function of $f$ .
<code>args</code>	List of additional arguments for optimization.
<code>h</code>	Hessian function of $f$ .

### Details

The argument `args` should be a list constructed from one of the following functions:

- `bfgs_args` for BFGS;
- `lbfgsb_args` for L-BFGS-B;
- `cg_args` for CG;
- `neldermead_args` for Nelder-Mead;
- `nlm_args` for the Newton-type algorithm used in `nlm`.

When `g` or `h` are omitted, the gradient or Hessian will be respectively be computed via finite differences.

### Value

A list with results corresponding to the specified function. See the package vignette for further details.

- `cg1` and `cg2` return a `cg_result` which is documented in the section "Conjugate Gradient".
- `bfgs1` and `bfgs2` return a `bfgs_result` which is documented in the section "BFGS".
- `lbfgsb1` and `lbfgsb2` return a `lbfgsb_result` which is documented in the section "L-BFGS-B".
- `neldermead` returns a `neldermead_result` which is documented in the section "Nelder-Mead".
- `nlm1`, `nlm2`, and `nlm3` return a `nlm_result` which is documented in the section "Newton-Type Algorithm for Nonlinear Optimization".

**Examples**

```
f = function(x) { sum(x^2) }
g = function(x) { 2*x }
h = function(x) { 2*diag(length(x)) }
x0 = c(1,1)
```

```
args = cg_args()
cg1(x0, f, g, args)
cg2(x0, f, args)
```

```
args = bfgs_args()
bfgs1(x0, f, g, args)
bfgs2(x0, f, args)
```

```
args = lbfgsb_args()
lbfgsb1(x0, f, g, args)
lbfgsb2(x0, f, args)
```

```
args = neldermead_args()
neldermead(x0, f, args)
```

```
args = nlm_args()
nlm1(x0, f, g, h, args)
nlm2(x0, f, g, args)
nlm3(x0, f, args)
```

---

outer

*Outer Matrix*

---

**Description**

Compute "outer" matrices and matrix-vector products based on a function that operators on pairs of rows. See details.

**Usage**

```
outer1(X, f)
```

```
outer2(X, Y, f)
```

```
outer1_matvec(X, f, a)
```

```
outer2_matvec(X, Y, f, a)
```

**Arguments**

X                    A numerical matrix.

$f$	Function $f(x, y)$ that operates on a pair of rows. Depending on the context, rows $x$ and $y$ are both rows of $X$ , or $x$ is a row from $X$ and $y$ is a row from $Y$ .
$Y$	A numerical matrix.
$a$	A scalar vector.

### Details

The `outer1` function computes the  $n \times n$  symmetric matrix

$$\text{outer1}(X, f) = \begin{bmatrix} f(x_1, x_1) & \cdots & f(x_1, x_n) \\ \vdots & \ddots & \vdots \\ f(x_n, x_1) & \cdots & f(x_n, x_n) \end{bmatrix}$$

and the `outer1_matvec` operation computes the  $n$ -dimensional vector

$$\text{outer1\_matvec}(X, f, a) = \begin{bmatrix} f(x_1, x_1) & \cdots & f(x_1, x_n) \\ \vdots & \ddots & \vdots \\ f(x_n, x_1) & \cdots & f(x_n, x_n) \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}.$$

The `outer2` operation computes the  $m \times n$  matrix

$$\text{outer2}(X, Y, f) = \begin{bmatrix} f(x_1, y_1) & \cdots & f(x_1, y_n) \\ \vdots & \ddots & \vdots \\ f(x_m, y_1) & \cdots & f(x_m, y_n) \end{bmatrix}$$

and the `outer2_matvec` operation computes the  $m$ -dimensional vector

$$\text{outer2\_matvec}(X, Y, f, a) = \begin{bmatrix} f(x_1, y_1) & \cdots & f(x_1, y_n) \\ \vdots & \ddots & \vdots \\ f(x_m, y_1) & \cdots & f(x_m, y_n) \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}.$$

### Value

`outer1` and `outer2` return a matrix. `outer1_matvec` and `outer2_matvec` return a vector. See section "Outer" of the package vignette for details.

### Examples

```
set.seed(1234)
f = function(x,y) { sum( (x - y)^2 ) }
X = matrix(rnorm(12), 6, 2)
Y = matrix(rnorm(10), 5, 2)
outer1(X, f)
outer2(X, Y, f)

a = rep(1, 6)
b = rep(1, 5)
outer1_matvec(X, f, a)
```

```
outer2_matvec(X, Y, f, b)
```

---

```
solve_cg
```

---

*Iteratively Solve a Linear System with Conjugate Gradient*

---

### Description

Solve the system  $l(x) = b$  where  $l(x)$  is a matrix-free representation of the linear operation  $Ax$ .

### Usage

```
solve_cg(l, b, init, args)
```

### Arguments

<code>l</code>	A linear transformation of $x$ .
<code>b</code>	A vector.
<code>init</code>	Initial value of solution.
<code>args</code>	List of additional arguments from <code>cg_args</code> .

### Value

A list with the form of a `solve_cg_result` described in section "Conjugate Gradient" of the package vignette.

### Examples

```
set.seed(1234)

n = 8
idx_diag = cbind(1:n, 1:n)
idx_ldiag = cbind(2:n, 1:(n-1))
idx_udiag = cbind(1:(n-1), 2:n)
b = rep(1, n)

## Solution by explicit computation of solve(A, b)
A = matrix(0, n, n)
A[idx_diag] = 2
A[idx_ldiag] = 1
A[idx_udiag] = 1
solve(A, b)

## Solve iteratively with solve_cg
f = function(x) { A %*% x }
args = cg_args()
init = rep(0, n)
solve_cg(f, b, init, args)
```

trunc

*Functions for Truncated Distributions***Description**

Density, CDF, quantile, and drawing functions for a univariate distribution with density  $f$ , cdf,  $F$ , and quantile function  $F^{-1}$  truncated to the interval  $[a, b]$ .

**Usage**

```
d_trunc(x, lo, hi, f, F, log = FALSE)
p_trunc(x, lo, hi, F, lower = TRUE, log = FALSE)
q_trunc(p, lo, hi, F, Finv, lower = TRUE, log = FALSE)
r_trunc(n, lo, hi, F, Finv)
```

**Arguments**

x	Vector of quantiles.
lo	Vector of lower limits.
hi	Vector of upper limits.
f	Density function with form $f(x, \log)$ .
F	CDF function with signature $F(x, \text{lower}, \log)$ , where $x$ is numeric and $\text{lower}$ and $\log$ are logical.
log	logical; if TRUE, probabilities are given on log-scale.
lower	logical; if TRUE, probabilities are $P(X \leq x)$ ; otherwise, $P(X > x)$ .
p	Vector of probabilities.
Finv	Quantile function with signature $\text{Finv}(x, \text{lower}, \log)$ , where $x$ is numeric and $\text{lower}$ and $\log$ are logical.
n	Number of draws.

**Value**

Vector with results. `d_trunc` computes the density, `r_trunc` generates random deviates, `p_trunc` computes the CDF, and `q_trunc` computes quantiles.

**Examples**

```
library(tidyverse)

m = 100 ## Length of sequence for density, CDF, etc
shape1 = 5
shape2 = 2
```

```

lo = 0.5
hi = 0.7

# Density, CDF, and quantile function for untruncated distribution
ff = function(x, log) { dbeta(x, shape1, shape2, log = log) }
FF = function(x, lower, log) { pbeta(x, shape1, shape2, lower.tail = lower, log = log) }
FFinv = function(x, lower, log) { qbeta(x, shape1, shape2, lower.tail = lower, log = log) }

# Compare truncated and untruncated densities
xseq = seq(0, 1, length.out = m)
lo_vec = rep(lo, m)
hi_vec = rep(hi, m)
f0seq = ff(xseq, log = FALSE)
fseq = d_trunc(xseq, lo_vec, hi_vec, ff, FF)
data.frame(x = xseq, f = fseq, f0 = f0seq) %>%
  ggplot() +
  geom_line(aes(xseq, fseq)) +
  geom_line(aes(xseq, f0seq), lty = 2) +
  xlab("x") +
  ylab("Density") +
  theme_minimal()

# Compare truncated densities and empirical density of generated draws
n = 100000
lo_vec = rep(lo, n)
hi_vec = rep(hi, n)
x = r_trunc(n = n, lo_vec, hi_vec, FF, FFinv)
hist(x, probability = TRUE, breaks = 15)
points(xseq, fseq)

# Compare empirical CDF of draws with CDF function
Femp = ecdf(x)
lo_vec = rep(lo, m)
hi_vec = rep(hi, m)
Fseq = p_trunc(xseq, lo_vec, hi_vec, FF)
data.frame(x = xseq, FF = Fseq) %>%
  mutate(F0 = Femp(x)) %>%
  ggplot() +
  geom_line(aes(xseq, FF), lwd = 1.2) +
  geom_line(aes(xseq, F0), col = "orange") +
  xlab("x") +
  ylab("Probability") +
  theme_minimal()

# Compare empirical quantiles of draws with quantile function
pseq = seq(0, 1, length.out = m)
lo_vec = rep(lo, m)
hi_vec = rep(hi, m)
Finvseq = q_trunc(pseq, lo_vec, hi_vec, FF, FFinv)
Finvemp = quantile(x, prob = pseq)
data.frame(p = pseq, Finv = Finvseq, Finvemp = Finvemp) %>%
  ggplot() +
  geom_line(aes(pseq, Finv), lwd = 1.2) +

```

```
geom_line(aes(pseq, Finvemp), col = "orange") +  
xlab("p") +  
ylab("Quantile") +  
theme_minimal()
```

---

univariate-optimization

*Univariate Optimization*

---

## Description

Univariate Optimization

## Usage

```
goldensection(f, lower, upper, args)
```

```
optimize_brent(f, lower, upper, args)
```

## Arguments

f	Function to optimize.
lower	Lower limit of search interval. Must be finite.
upper	Upper limit of search interval. Must be finite.
args	List of additional arguments from the function optimize_args.

## Value

A list with the form of a `optimize_result` described in section "Univariate Optimization" of the package vignette.

## Examples

```
f = function(x) { x^2 - 1 }  
args = optimize_args()  
goldensection(f, 0, 10, args)  
optimize_brent(f, 0, 10, args)
```



---

which0	<i>Matrix Which Function</i>
--------	------------------------------

---

**Description**

Matrix Which Function

**Usage**

```
which0(X, f)
```

**Arguments**

X	A matrix
f	A predicate to apply to each element of $X$ .

**Details**

The which C++ functions are intended to operate like the following call in R.

```
which(f(X), arr.ind = TRUE) - 1
```

The R functions exposed here are specific to numeric-valued matrices, but the underlying C++ functions are intended to work with any type of Rcpp Matrix.

**Value**

A matrix with two columns. Each row contains a row and column index corresponding to an element of  $X$  that matches the criteria of  $f$ . See section "Which" of the package vignette for details.

**Examples**

```
X = matrix(1:12 / 6, nrow = 4, ncol = 3)
f = function(x) { x < 1 }
which0(X, f)
```

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